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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DESIGN OF AN OPEN-CHANNEL CONTROL SECTION

By Karl R. Kennison,[1] M. Am. Soc. C. E.

Synopsis

In designing an open channel to measure the flow of water, the shape of the controlling section to produce any desired rating curve can be determined by a definite mathematical relationship, the expression of which is claimed to be new, and to have possibilities in the application to practical problems. An analysis of the problem is contained in this paper. For example, if it is desired to modify a Venturi flume so that the rating curve at the given piezometer section immediately up stream from the controlling throat will be a predetermined convenient straight line, it can be done readily by the method presented herein. The writer first outlines the principles involved and arrives at mathematical expressions for the shape of the controlling section in terms of the desired rating, and then illustrates the application by the solution of specific problems.

Introduction

In any weir, flume, open nozzle, etc., it does not appear to be difficult to fit the dimensions of the controlling section to the desired rating curve by cut-and-try computations. However, the writer has frequently made designs of this character and has been intrigued by the observation that, even with precision so extreme that it seems unjustifiable, such cut-and-try computations do not give the true theoretical dimensions of the control. For example, at any depth of flow there may be identical velocities, with a great variety of section shapes, as long as the section areas are the same, excess width at one point being offset by decreased width at another. In fact, two controlling sections can be quite different in general appearance; that is, they can have a different relationship between depth and width and yet their two curves of area, plotted against depth, can be so nearly alike that the resulting rating curves are the same within the precision expected in hydraulic computations.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted before **September 15, 1939.**

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For this reason, cut-and-try computations fail to reveal the true theoretical shape of a controlling section required to produce a given rating curve, although they may solve any single specific problem with entire satisfaction so far as practical results are concerned.

The writer has long felt the need of a better understanding of the principles involved. His earlier attempts to obtain a correct analytic solution of the problem were discouraged by the mathematical complications encountered. A solution is presented herein which obtains the desired result—namely, simple expressions for the width and depth of the controlling section in terms of the given rating.

Theoretical Solution

Referring to Fig. 1, it is required to find d_c and B_c in terms of the desired rating curve—that is, in terms of Q, d_p, S and the elements of the given piezometer section, or combinations of them, B_p, A_p, and H_p. The frictional loss, h_f, is relatively quite small and the first assumption in the interest of

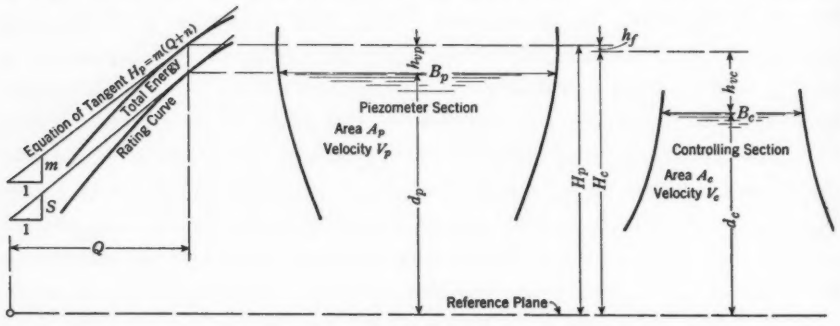

Fig. 1

practical simplicity is to neglect it and to assume H_p equal to H_c in finding the expressions for d_c and B_c. It is practical as well as accurate and convenient to do this and to take the relatively small amount of friction into account later when finding the actual dimensions in any specific problem.

Three general equations, applicable to any conditions and variations of flow, through any type of section, even through a section whose dimensions themselves are variable, are as follows:

$$Q = A_c V_c \quad \ldots\ldots (1a)$$

$$V_c = \sqrt{2 g h_{vc}} \quad \ldots\ldots (1b)$$

and,

$$h_{vc} = H_p - d_c \quad \ldots\ldots (1c)$$

A fourth general formula—

$$H_p = m (Q + n) \quad \ldots\ldots (2)$$

—requires the condition of critical flow through a given controlling section. Equation (2) also becomes general if m' and n' are substituted for m and n, such that

$$H_p = m' (Q + n') \quad \ldots\ldots (3)$$

Combining Equations (1) and (3):

$$\frac{Q^2}{2\,g\,A_c^2} = m'\,Q + m'\,n' - d_c \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

There are five variables in this general equation. To expedite the analysis, consider a continuous variation of flow, in a section of given dimensions, under a constant head H_p. When the desired critical depth d_c is reached, the discharge Q is at its maximum rate; and, m' and n' become m and n, which are constant because H_p is constant. Furthermore, in any given section the area varies with the depth and *vice versa*, and for present purposes one may assume d_c to be some function of A_c. Hence dd_c = (function of A_c) dA_c.

Accordingly, to determine the desired relationships in a controlling section, differentiate Equation (4), substituting m and n as constants instead of m' and n' and (function of A_c) dA_c instead of dd_c. As a result:

$$\frac{dQ}{dA_c} = \frac{Q^2 - f\,(A_c)}{A_c\,(Q - A_c^2\,g\,m)} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Since Q is a maximum, $\frac{dQ}{dA_c}$ can be equated to 0 so that the relation between Q and A_c is

$$Q = A_c^2\,g\,m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Again, substituting m and n for m' and n' in Equation (4) and combining it with Equation (6), one finds, by eliminating A_c, that

$$d_c = \frac{m\,Q}{2} + m\,n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and, by eliminating Q, that

$$A_c^2\,m^2\,g = 2\,(d_c - m\,n) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Since B_c is the differential of A_c with respect to d_c, one can differentiate Equation (8), m and n continuing constant as before; thus:

$$\frac{dA_c}{dd_c} = B_c = \frac{1}{A_c\,m^2\,g} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Combining Equation (9) with Equation (6) to eliminate A_c:

$$B_c = \frac{1}{\sqrt{m^3\,Q\,g}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Thus Equations (7) and (10) are simple expressions for the depth and width at the controlling section, in terms of m and n; and the latter may be expressed in terms of the given rating curve and the elements of the given piezometer section as follows:

Since velocity head varies as the square of velocity, its rate of change is twice that of the velocity; and, since velocity varies as quantity discharged and inversely as area, its rate of change is the difference between those of the

quantity and of the area. In other words

$$\frac{dh_{vp}}{h_{vp}} = \frac{2\,dV_p}{V_p} = 2\left(\frac{dQ}{Q} - \frac{dA_p}{A_p}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

It is obvious that $dA_p = B_p\,dd_p$, and hence

$$\frac{dA_p}{dQ} = B_p\frac{dd_p}{dQ} = B_p\,S \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Since, by definition, $H_p = d_p + h_{vp}$; $\frac{dH_p}{dQ} = m$; and, $\frac{dd_p}{dQ} = S$:

$$m = S + \frac{dh_{vp}}{dQ} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Combining Equations (11), (12), and (13) to eliminate the differentials:

$$m = S + 2\,h_{vp}\left(\frac{1}{Q} - \frac{B_p\,S}{A_p}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Also, by definition:

$$n = \frac{H_p}{m} - Q \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Thus, the values of the convenient intermediaries m and n are given in Equations (14) and (15), in terms of the desired rating curve and the known elements of the given piezometer section, which values of m and n may be substituted in Equations (7) and (10) to determine the precise theoretical shape of the controlling section necessary to produce the desired rating.

Application of Theory to Practice

It is obvious that the foregoing computations are based on a theoretically perfect condition, free from cross currents and eddies, in which the filaments of flow are horizontal, and the pressure proportional to the depth. A satisfactory condition in this respect will not be obtained at the throat of the controlling section if the latter is too short or its immediate approach too suddenly contracted. It is not the purpose of this paper to discuss the proper design of a contraction to produce a smooth horizontal flow. A sufficiently gradual section change to a throat or crest which is not too short in the direction of flow, together with a sufficient length of the critical throat section to straighten the filaments, will result in a condition of flow past the controlling section which approaches remarkably close to the perfection sought. For example, in the writer's experience, a properly designed, flat-crested, submerged weir is an extremely accurate measuring device and one which shows little if any difference between theoretical and actual rates of discharge. However, it is not the intention to disclaim the practical necessity of modifying the theoretical relationships by an empirical coefficient, since the principal purpose of this paper is to analyze and present these relationships.

Correction for Friction

This practical necessity for an appreciable length of water travel between the piezometer section and the controlling section results in a certain fric-

tional loss between the two. Such loss is relatively small and may be compensated for by constructing the entire measuring device on a slight slope. It is generally so small that rough computations and estimates of the weighted average hydraulic elements and coefficient of roughness will indicate the appropriate slope to be given to the invert or reference plane of the measuring device.

Although such a fixed slope will generally be sufficient to compensate for frictional losses, throughout the desired range of discharge rates, without material sacrifice of precision, a more complete and very precise compensation can best be made as follows: The foregoing rough computations and estimates will give the frictional loss or slope of the total-energy gradient between the piezometer section and the controlling section for different rates of discharge. In a device of the type under discussion, this slope will be fairly constant throughout a considerable range of the lower rates of discharge but will increase noticeably at the higher rates. Hence one may select, arbitrarily, a fixed slope for the invert or reference plane which will compensate for the frictional loss at the lower rates of discharge, so that only at the higher rates of discharge must the extra head loss, h_f, or difference between H_p and H_c, be accounted for. The easiest and most satisfactory way to do this is to plot a curve of the roughly computed values of h_f for different values of Q from which the slope, $\frac{dh_f}{dQ}$, as well as h_f, can be read for any value of Q. To make the correction for excess frictional loss, subtract h_f from H_p and $\frac{dh_f}{dQ}$ from m. Equation (14) then becomes

$$m = S + 2\,h_{vp}\left(\frac{1}{Q} - \frac{B_p\,S}{A_p}\right) - \frac{dh_f}{dQ} \quad \ldots\ldots\ldots\ldots (16)$$

and Equation (15) becomes

$$n = \frac{H_p - h_f}{m} - Q \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

If friction is neglected (that is, if one does not wish to use expressions to which corrections for excess frictional loss at the higher rates of flow can be applied as described herein) Equations (7), (10), (14), and (15) can be combined to obtain the dimensions of the critical section in terms of the desired rating curve and of the known elements of the given piezometer section, without using the values of m and n. Thus, for the drop in water level between the piezometer section and the controlling section:

$$d_p - d_c = Q\,S\left(\frac{1}{2} - \frac{h_{vp}\,B_p}{A_p}\right) \quad \ldots\ldots\ldots\ldots\ldots (18)$$

The width of the controlling section can be expressed similarly, but the equation would be too complicated to warrant its use instead of Equations (10) and (14).

Illustrative Examples

Example 1.—In Fig. 2, a piezometer section is chosen arbitrarily, as an example, with a 3-ft bottom width, a 3-ft depth and side slopes of 4 on 1.

The rating curve is also chosen arbitrarily as a straight line, the maximum discharge being 28.3 cu ft per sec. Of course, such a line must show, at all points, a definitely greater depth, less discharge, than could pass the piezometer section itself as a controlling section. Otherwise, because of the well-understood tendency toward the propagation of surface waves, a definite relation between rate of discharge and depth at the piezometer section would not be assured at the lesser depths. To insure this condition the invert, in this example, is arbitrarily raised 0.1 ft at the controlling section (see Fig. 2), and the straight-line rating is drawn tangent to the rating curve which this raised invert causes at these lesser depths.

TABLE 1.—SUMMARY OF COMPUTATIONS TO DETERMINE DIMENSIONS OF CONTROLLING SECTION

(Ten-Inch Slide-Rule Computations)

Rate of discharge, Q, in cubic feet per second	Piezometer Section					Slope, m, of the total-energy rating curve	Values of n†	Controlling Section	
	Depth, d_p, in feet*	Area, A_p, in square feet*	Surface width, B_p, in feet	Velocity head, h_{vp}, in feet	Energy head, H_p; total energy gradient*			Depth, d_c, in feet*	Surface width, B_c, in feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) S = 0.0982; STRAIGHT-SIDED CHANNEL SHOWN IN FIG. 2									
28.30	3.00	11.25	4.50	0.098	3.098	0.0974	3.51	1.720	1.092
23.21	2.50	9.06	4.25	0.102	2.602	0.0976	3.47	1.470	1.200
18.12	2.00	7.00	4.00	0.104	2.104	0.0980	3.35	1.216	1.350
14.04	1.60	5.44	3.80	0.103	1.703	0.0988	3.20	1.009	1.519
9.97	1.20	3.96	3.60	0.098	1.298	0.1004	2.95	0.797	1.757
5.89	0.80	2.56	3.40	0.082	0.882	0.1047	2.53	0.574	2.146
2.00	0.418	1.298	3.209	0.0368	0.455	0.1171	1.88	0.337	3.112
(b) S = 0.0924; CIRCULAR OPEN NOZZLE SHOWN IN FIG. 3									
28.30	3.00	7.069	0	0.248	3.248 3.233‡	0.1099	1.275	1.694	0.909
25.05	2.70	6.701	1.800	0.216	2.916 2.906‡	0.0990 0.0980‡	4.42 4.60‡	1.677 1.678‡	1.131 1.150‡
20.18	2.25	5.687	2.598	0.195	2.445 2.439‡	0.0952 0.0945‡	5.50 5.62‡	1.484 1.485‡	1.335 1.350‡
15.30	1.80	4.428	2.939	0.185	1.985 1.982‡	0.0939 0.0934‡	5.85 5.92‡	1.267 1.268‡	1.566 1.580‡
10.43	1.35	3.085	2.985	0.177	1.527 1.526‡	0.0947 0.0944‡	5.70 5.74‡	1.033 1.034‡	1.872 1.880‡
7.18	1.05	2.205	2.861	0.164	1.214	0.0987	5.11	0.859	2.118
4.03	0.759	1.405	2.508	0.128	0.887	0.1137	3.77	0.658	2.290

* Measured above the fixed reference plane.
† Intercept in the equation of the tangent to the total-energy rating curve, such that $H_p = m(Q + n)$.
‡ Includes correction for excess friction.

Table 1 is a summary of computations made to determine the shape of the controlling section, m being computed from Equation (14), n from Equation (15), d_c from Equation (7) or directly from Equation (18), and B_c from Equa-

tion (10). The shape of the controlling section is plotted from the computed values of d_c and B_c, with the results shown in Fig. 2. (It should be noted that the Sutro weir[2] is a special case of the general case presented in this paper.)

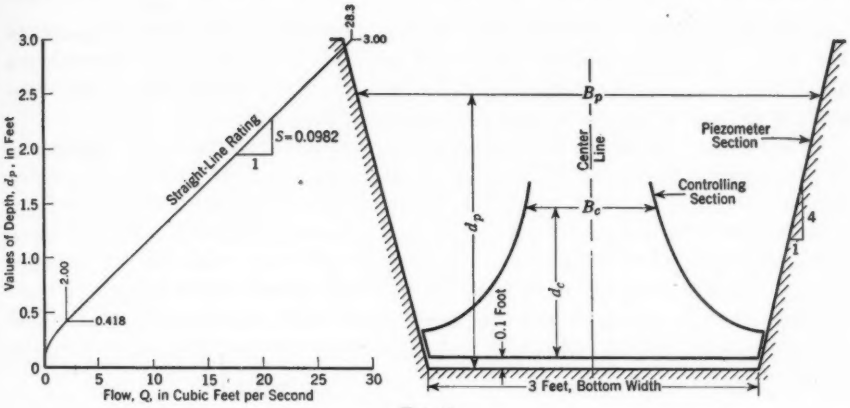

FIG. 2

Example 2.—Another example is modeled after the so-called open nozzle[3] for measuring the flow of sewage. As shown in Fig. 3(*b*), the piezometer section is circular, 3 ft in diameter. The contraction of the invert of the controlling section is made arbitrarily by using a center offset of 0.075 ft (see Fig. 3). As in Example 1, the rating curve is an arbitrary straight line, the maximum discharge being 28.3 cu ft per sec.

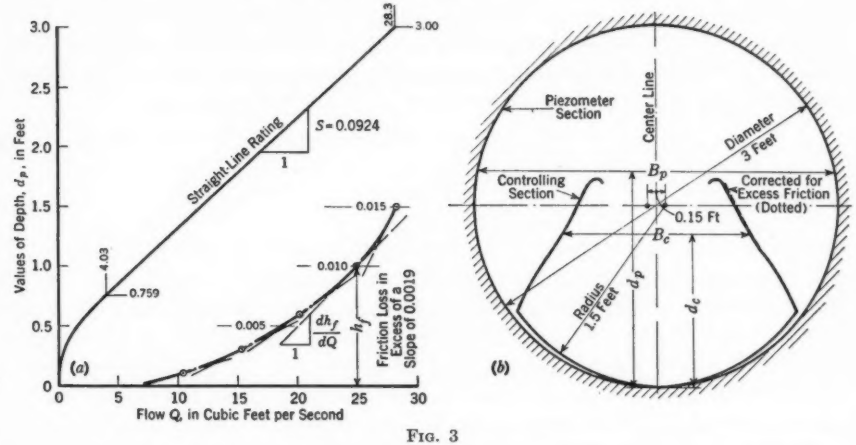

FIG. 3

To illustrate the application of the correction for excess friction, as well as its smallness, the velocities and hydraulic elements were computed at different depths in the case of this open nozzle. They indicated that the invert or

[2] "Sutro Weir Investigations Furnish Discharge Coefficients," by F. T. Mavis, M. Am. Soc. C. E., E. Soucek, Jun. Am. Soc. C. E., and H. E. Howe, *Engineering News-Record*, November 12, 1936, p. 679.

[3] *Journal*, Boston Society of Civil Engineers, January, 1934.

reference plane should be on a slope of about 0.0019 to compensate for the frictional losses at the lesser depths, and that at the greater depths the excess friction, h_f, to be subtracted from H_p, and its rate of change, $\frac{dh_f}{dQ}$, to be subtracted from m, are approximately as shown by the supplementary graph in Fig. 3(*a*). These corrections have been applied as shown in the parentheses in Table 1 and result only in a slight widening of the upper part of the controlling section, as shown by the dotted line in Fig. 3(*b*).

The exact shape of the controlling section becomes relatively less important as the depth d_c increases. The result is that, although a circular piezometer section, closed at the top, results in the peculiar theoretical shape of the top of the controlling section shown in Fig. 3(*b*), the omission of this sharp irregularity, and the substitution of any convenient arbitrary extension of the sides of the controlling section, merely results in a negligible gradual departure from the straight-line rating at depths greater than 95% of the diameter. The maximum departure at full pipe is only about one-third of 1% increase in the rate of discharge.

Conclusion

The depth and width at the controlling section given by Equations (7), (10), and (18) are those which, if adhered to throughout the entire range of discharge, or depth, will result in the desired rating. This does not mean that in any particular limited case another set of dimensions determined, for example, by an approximate cut-and-try computation, will not fill the specifications satisfactorily. However, the section dimensioned by these formulas is the only one that will be theoretically correct throughout the entire range of depths; and, a knowledge of the theoretically correct shape cannot fail to contribute to good design in the selection of dimensions to fit the conditions in any case. There are undoubtedly other adaptations than those illustrated herein.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

FLASH-BOARD PINS

By Chilton A. Wright,[1] and Clifford A. Betts,[2]
Members, Am. Soc. C. E.

Synopsis

Formulas and stress values for the design of pipe supports on the automatic flash-board gates of dam spillways are developed and discussed in this paper. As a result of hydraulic laboratory tests of full-size flash-boards, supplemented by mechanical tests of the pipe supports and check tests of actual field installations outlined herein, data have been obtained which permit accurate predetermination of the lake level at which gates of this type will automatically fall and by-pass flood water.

In addition to the description of the tests and the analysis of the relations between water head, height of flash-boards, and size and spacing of pipe, the results have been consolidated to serve as a guide in the design of flash-board installations. On this basis the United States Forest Service has designed hinged flash-board gates, supported by steel pipes, and has installed them on a number of their dams. Field tests, made at these dams later, yielded results that closely check the laboratory tests.

Flash-Boards

Automatic flash-board gates are a logical development of the conventional flash-board for reservoir water-surface regulation. Being a structure that increases the height of a dam, usually on the spillway section (see Fig. 1), the flash-board is designed to fail under a predetermined head of water and fall, thereby increasing the flood-carrying capacity of the spillway. In the past many flash-board installations have been constructed, using light, inexpensive planks, supported by closely spaced pins, and were lost after failure during a flood. Many other types of flash-boards have been used in which failure was induced mechanically by cutting anchor wires or posts or by lifting with traveling hoists.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 15, 1939.**

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Conservation of the storage at the top of a reservoir, where every foot of additional depth represents the cream of the storage, is of high economic value during dry periods, particularly on water-power and water-supply projects. On numerous streams where spring floods are followed by extended periods of low flow, flash-boards can be raised toward the end of the high-water stage and the storage can be augmented for carrying through the drought period. Control of water-surface fluctuations within narrow limits is advantageous and sometimes imperative for fish and game lakes, as well as on recreational developments. As a "safety valve" against back-water overflow or sustained inundation, flash-boards are distinctly effective. In arid regions conservation of water usually requires hand gates, but in humid regions where the flood menace, rather than the water loss, is the prime consideration, the cost of hand-operated flood gates is not justified when the same function can be performed by simple, automatic flash-board gates.

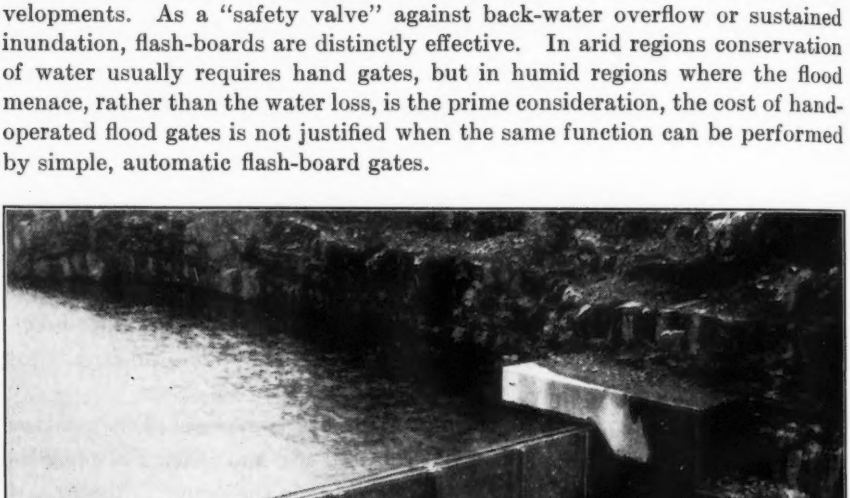

FIG. 1.—HINGED AUTOMATIC FLASH-BOARD GATE, 2.5 FEET HIGH BY 35 FEET LONG, AT LAKE SHERANDO DAM, GEORGE WASHINGTON NATIONAL FOREST

The cost of providing flash-boards is small compared with that of increasing the size of the spillway. In many cases it is almost prohibitive to provide spillway capacity, adequate to handle rare yet disastrous floods, unless flash-boards are used. For the control of large floods, a deep spillway gate can be constructed in multiple panels which will fall successively, thereby limiting the discharge in the channel below, except for exceptional floods, and using the pondage to equalize the flow.

Flash-boards of plank may be converted into flash-board gates by the addition of hinges and stiffening members of wood or steel. The entire structure can be made of wood, steel, or other structural material. After the flood the gates may simply be raised again and the broken supports replaced.

By providing dependable regulation at low cost, thereby protecting not only the dam but also the property below and the lake shores above, the automatic flash-board gate becomes an important factor in water conservation.

Necessity for Tests

Such use of flash-boards presupposes failure of the supporting pipe within narrow limits of reservoir water level. Positive failure of the flash-boards is essential to successful operation, inasmuch as the entire dam is in danger if the flash-boards do not function as designed. Region Seven of the U. S. Forest Service had the problem of providing flash-boards for a number of earth dams similar to that shown in Fig. 1, where certain action of the flash-boards was essential.

A search of the published literature[3] for data to be used in the general design of supports for flash-board installations yielded only a few isolated tests of pins, made during construction, and gave no assurance that the pins or pipes would fail reasonably near the desired water head. In fact, discrepancies of 2 ft, or 50%, were obtained by using stresses suggested by different writers.

Thus, tests were required to furnish dependable design data. The Forest Service suggested and co-operated in a series of tests made in the National Hydraulic Laboratory, National Bureau of Standards, on various types and sizes of flash-boards, mounted on a low dam crest, and supported by pins consisting of various sizes of standard commercial steel and wrought iron pipe. A complete report of these tests was prepared under date of April, 1937 (15). A copy of this report has been filed for reference in the Engineering Societies Library.[4] A brief summary of the test results was published by one of the writers in 1936 (16).

Scope of Investigation

Hydraulic Tests.—The general investigation is divided into three distinct parts: (a) The experimental determination of the modulus of rupture for the pipes used to support the flash-boards; (b) the use of these moduli of rupture to compute the head or water level at which various flash-board installations should fail, using various simplified moment formulas for this purpose; and (c) the comparison of these computed heads with the heads at which failure actually occurs in various flash-board installations in order to determine the accuracy with which the various moment formulas can be used.

The hydraulic tests were made of pipe ranging from $\frac{3}{4}$ in. to 3 in. in size (see Table 1). Spacing between the pipes was varied from 5 to 8 ft. The wider spacing of the larger pipes was found necessary in order that they might collapse under the limited water head available in the laboratory flume. The flash-boards ranged from 1 ft 10 in. to 5 ft 2 in. in height above the dam crest. In addition to measuring the water elevations above and below the dam, and deflections of the pipe under load, its behavior prior to, and after, the yield point was reached was observed. Panels about 3.5 ft by 5 ft having "V" joints and "butt" joints were tried before concentrating the tests on continuous flash-boards fastened together to operate as a single unit.

The moduli of rupture were computed from these tests and formulas developed for design. Then, as a ready means of checking the formulas, the laboratory set-up was assumed to be a field installation, the formulas were

[3] See bibliography in Appendix I. Line numbers in parentheses are used to indicate references.

[4] 33 West Thirty-Ninth Street, New York, N. Y.

applied thereto, and their accuracy was judged by comparison with the test results.

Mechanical Tests.—In order to determine whether simple mechanical tests could be substituted for the more elaborate hydraulic tests and still yield the

TABLE 1.—DIMENSIONS FOR NESTING PIPE

Pipe size, in inches	Pipe Diameter,* in Inches					
	Standard		Extra Heavy		Sleeves	
	External	Internal	External	Internal	External	Internal
0.75	1.050	0.824	1.050	0.742	1.315†	1.062†
1.00	1.315	1.049	1.315	0.957	1.900	1.610‡
1.25	1.660	1.380	....	....	1.900	1.610
1.50	1.900	1.610	1.900	1.500	2.375	2.067
2.00	2.375	2.067	2.375	1.939	2.875	2.469
2.50	2.875	2.469	2.875	2.323	3.500	3.068
3.00	3.500	3.068	3.500	2.900	4.000	3.548
3.50	4.000	3.548	4.000	3.364	4.500	4.026
4.00	4.500	4.026	4.500	3.826	....	....

* Standard diameters from Federal Specification WW-P-403 for steel pipe.
† Federal Specification WW-P-351 for brass pipe = iron pipe size; this sleeve is the only brass pipe.
‡ Lined with tubing.

equivalent necessary values of modulus of rupture, samples were cut from the pipes used in the hydraulic tests. These specimens were tested as cantilevers in a testing machine, and the results were compared with those derived from the hydraulic tests.

Pipe vs. Solid Pins.—Tests made on solid steel pins prior to these pipe tests had revealed certain inherent disadvantages of the solid type. A small variation in diameter, composition, or heat treatment made a relatively large change in load capacity—a striking contrast to the pipe behavior (from previous tests made for the Forest Service by the National Bureau of Standards). Furthermore, extensive tests on mild steel pins by the Pennsylvania Water and Power Company of Baltimore, Md., indicated that the solid pins continued to deflect and carry load until they had deflected nearly 90 degrees.

DESCRIPTION OF EXPERIMENTS

Hydraulic Tests

General Layout.—The hydraulic tests were made in the main concrete flume on the first floor of the National Hydraulic Laboratory (see Fig. 2). It is 12 ft by 12 ft in cross-section, and was supplied with water by three pumps having a combined capacity of 35 cu ft per sec. The pumps discharged into a concrete tank about 25 ft by 32 ft in section, at the upper end of the flume, as shown in Fig. 3. Two motor-operated gates, 6 ft square, were used to control the flow into the flume. During most of the tests, a bulkhead, built for use in another investigation, was present in the flume down stream from the experimental set-up, but finally it was removed to prevent the tail-water from rising too rapidly during the tests. A large drain was provided in the side of the flume down stream from the flash-boards to return the water to the supply basin below the floor.

About 52 ft down stream from the control gates, a concrete dam crest, 21 in. high and about 28 ft long, equivalent to the crest of an actual dam, was constructed across the flume. To obtain this length, it was necessary to place the crest diagonally as shown in Fig. 2. For the preliminary experiments, five flash-boards, each 5 ft long and 3 ft 6 in. high, were placed on the crest and were supported by pipes of the same diameter placed in sockets embedded in the crest of the dam at 5-ft intervals.

In these preliminary tests, because the larger pipes were too strong to fail under a static water load produced by a head-water level even with the top of the flash-boards, a fixed wooden bulkhead with its crest 9 ft 4 in. above the

FIG. 2.—HYDRAULIC LABORATORY FLUME

flume floor was provided for increasing the head-water level (see Figs. 3, 4 and 5). Dressed lumber (Douglas fir) $2\frac{5}{8}$ in. thick was used for both the bulkhead and flash-boards, and the horizontal joints were made water-tight by 1-in. splines between the boards. The flash-boards tested in Preliminary Series No. 1 had wedge-shaped "V" joints at the ends (see Fig. 3). In Preliminary Series No. 2, the flash-boards were provided with "butt" joints, or square ends, bound with rubber to reduce leakage as shown in Fig. 4. A rubber apron on the up-stream face of the bulkhead prevented water from discharging over the top of the flash-boards as they deflected during the test.

For the final series of tests the fixed bulkhead was removed, and a one-piece flash-board 16 ft long and of heights varying from 1 ft 10 in. to 5 ft 2 in. above the dam crest was constructed. It was supported by only two pipes, symmetri-

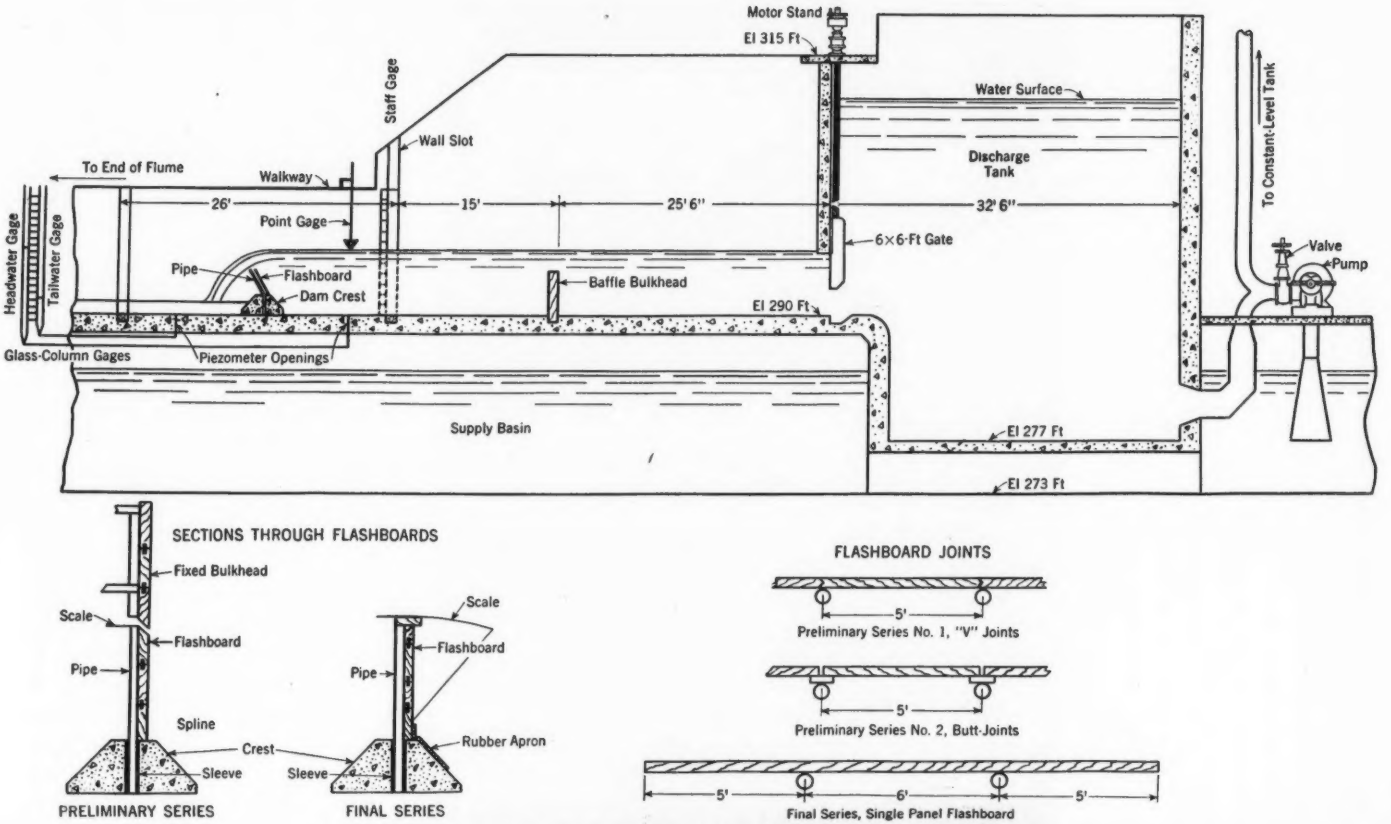

FIG. 3.—GENERAL LAYOUT: FLUME, TANK, BAFFLES, AND FLASH-BOARDS (NOT TO SCALE)

cally placed. The ends, bound with rubber, were closely fitted to wooden wing walls, as shown in Fig. 6. The rubber binding produced no appreciable friction on the end walls, and the flash-board could be moved very freely by hand before the test. One flash-board was balanced horizontally, and the distance of its center of gravity from the bottom was determined. This was used as a basis for computing the center of gravity of the other flash-boards. The lower part of the gate was braced, as can be seen in Fig. 2, to reduce the end deflections caused by water pressure.

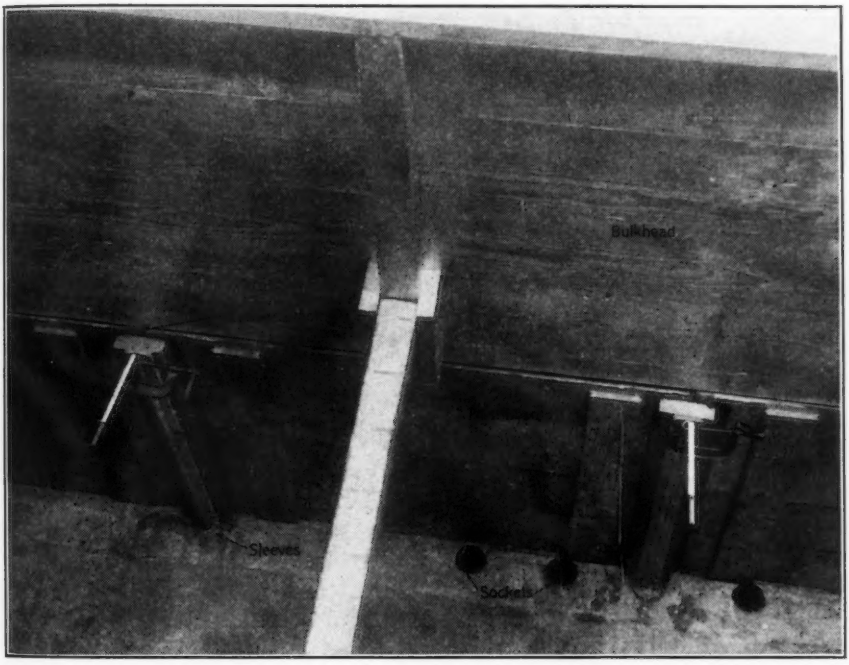

FIG. 4.—FLASH-BOARD PANELS WITH BUTT JOINTS

The various pipes tested were fitted into the 4-in. sockets in the crest by means of pipe sleeves of various diameters, 20 in. long, the sleeves being machined when necessary to make them nest together properly. The dimensions for nesting pipes and sleeves are given in Table 1 and the sleeves are shown in Fig. 7.

The external diameters of the test pipes were measured in three places: The bottom, top, and at the top of the supporting sockets. The pipes were weighed so that the average internal diameters could be computed. For pipes larger than 2 in., the internal diameter was also measured directly. Computed from the weight, they showed an average variation from directly measured values of 0.3% for the mechanical-test specimens and 0.1% for the eight hydraulic-test specimens compared. The pipes were set up in the sockets

and brought to a firm contact with the flash-boards by wooden shims when necessary.

Measuring Devices Used.—A scale, curved approximately to the arc of a circle, with the free length of the pipe as a radius, was fastened either on the top of the flash-board or to the pipe supporting the flash-boards (see Fig. 6). A transit mounted on top of the flume was used to read the scale as the pipe deflected. For later overflow tests, the pipe was extended up through the water to carry the scale (see Fig. 8). As a result of this there could be only a slight degree of vacuum under the nappe.

FIG. 5.—FLASH-BOARDS WITH "V" JOINTS AT FAILURE

Up stream from the center of the dam, as shown in Fig. 3, a brass piezometer connection, with an opening $\frac{5}{8}$ in. in diameter, was placed in the floor of the flume and led to a glass-column gage having an internal diameter of 1 in. A similar provision was made for reading the tail-water elevation but this was not used, owing to excessive turbulence in the water below the dam. For the overflow tests with large quantities of water, in which the head-water was also greatly disturbed, a plumb-bob or point gage was used as indicated in Fig. 3. A staff gage, graduated in 3-in. intervals, was painted on the steel bulkhead slot in the flume up stream from the dam for approximate observation of the head-water elevations.

Non-Overflow Tests.—The head-water was raised gradually by means of the pumps, and deflection readings were taken at 3-in. intervals, as determined by

the staff gage. Readings of the deflection, staff gage, and glass gage were taken simultaneously on signal. When the pipe began to deflect more rapidly, the readings were taken as the flash-board passed given deflection angles. When the pipe failed, the maximum readings of all quantities were taken. After failure, the pipes and flash-boards were examined and photographed. In some of the preliminary tests the flash-boards did not fail because the water flowed over the top of the bulkhead before creating sufficient head for failure. A view of Non-overflow Test No. 4, Preliminary Series No. 1, is shown in Fig. 5.

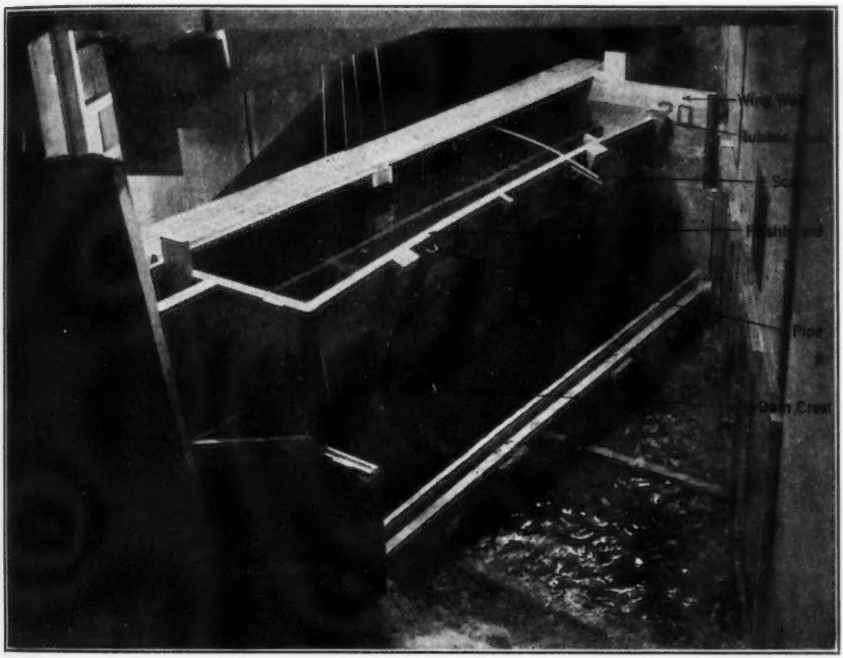

FIG. 6.—NON-OVERFLOW TEST NO. 20 AT FAILURE; 3-INCH PIPE

Overflow Tests.—The same long, single flash-board that was used for the non-overflow tests was used for these tests, and the pipes were also set up in the same manner. However, it was necessary to use a quite different experimental procedure in the overflow tests, because the tail-water rose so rapidly that it backed up against the down stream face of the flash-boards unless failure occurred quickly. Even after the obstruction down stream from the dam had been removed, the tail-water occasionally rose until its surface was above the level of the dam crest, particularly in tests of the higher flash-boards.

For some of these tests 35 cu ft per sec was not sufficient, and a method of utilizing the discharge tank was devised. The tank (see Fig. 3) was filled to a point near the top, and one 6-ft gate was opened slightly until the water began to flow over the baffle-bulkhead up stream from the flash-boards. As soon as the water appeared on the face of the flash-boards, simultaneous readings

were made and were repeated at every additional 3-in. height. When the water reached the top of the boards, the gate in the discharge tank was opened rapidly, and the head of the water flowing over the boards was brought quickly to the maximum that could be obtained. In this manner a large discharge was produced over the flash-boards—at times probably as much as 150 cu ft per sec. The deflection was observed closely and, at the instant of failure, readings were taken of all quantities. The failure was fairly clear-cut, in that the movement of the pipe accelerated gradually until a sudden collapse occurred. A view of

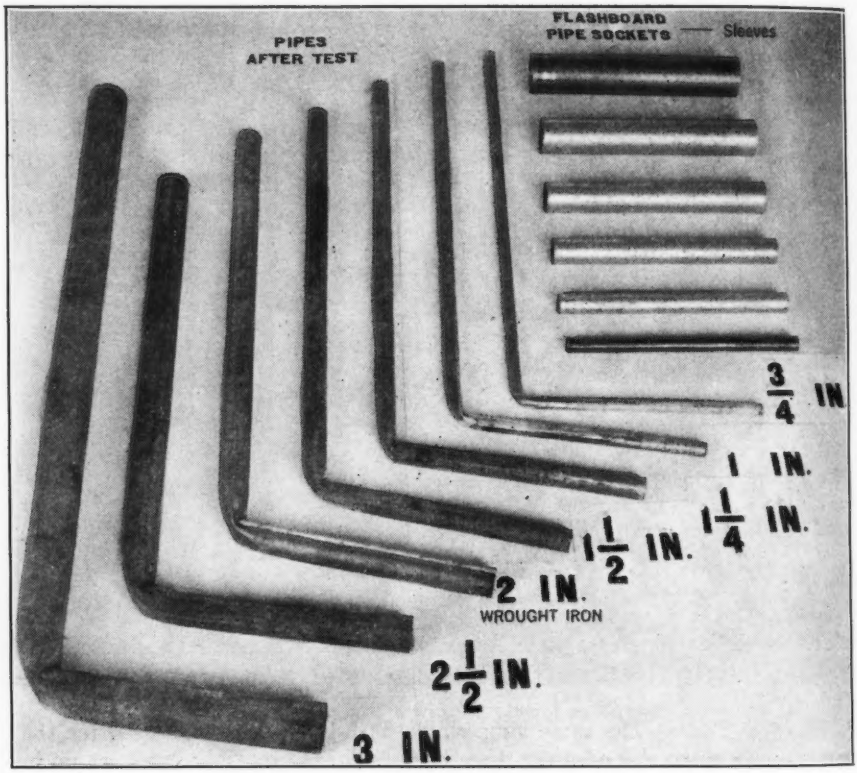

FIG. 7.—PIPES AND PIPE SLEEVES AFTER TESTS

Final-Series Overflow Test No. 27 at the point of failure is shown in Fig. 8. If the tail-water rose above the dam crest level (base of flash-board) before failure of the flash-board pipes, the flume was drained and the test repeated.

Fig. 7 shows the typical appearance of the pipes after failure. The bending was concentrated at the section of the pipe in the plane at the top of the socket: All of the pipes collapsed at this section, most of them splitting along the seam, and some across the bend.

Accuracy of Measurement.—For most of the tests the head was determined by means of the glass-column gages, but in the later overflow tests the point

gage was used. By comparative tests it was established that the head measurement was accurate to 0.01 ft for the non-overflow tests, 0.02 ft for most of the overflow tests, and 0.03 ft for tests with violent disturbance of the head-water.

Results of Hydraulic Tests.—The net result of the Preliminary Series No. 1 was to disclose that the indeterminate influence of the binding of the "V" joints was frequently as great as the supporting power of the pipe, and therefore such joints could not be tolerated. Preliminary Series No. 2 also showed sufficient binding in the 90° "butt" joints to interfere with the design of the flash-boards on a short spillway, although the effect would probably be insignificant

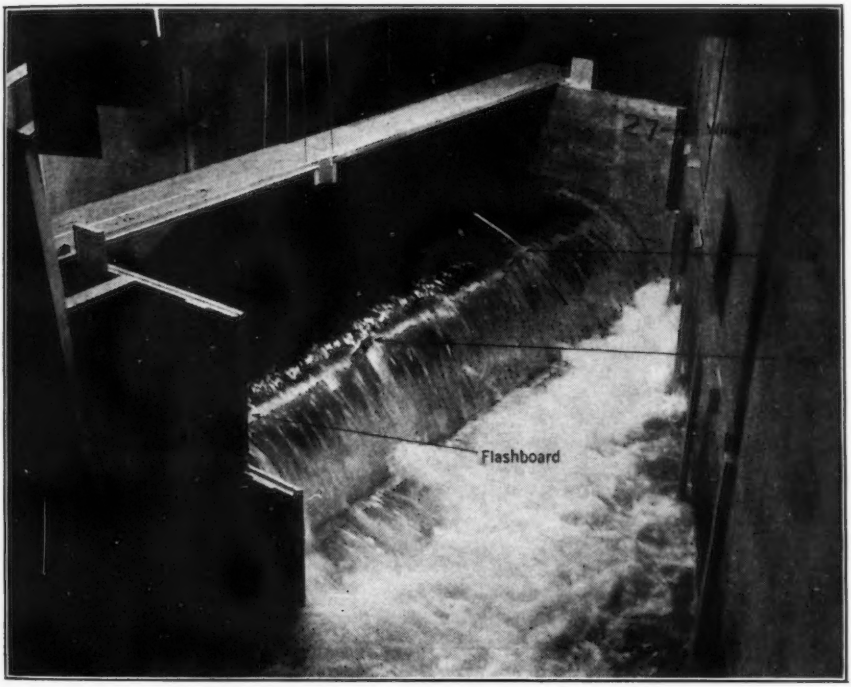

FIG. 8.—OVERFLOW TEST NO. 27 AT FAILURE; 1¼-INCH PIPE

on a long spillway. In both series the stresses in the pipes computed from the water loads were excessive.

Accordingly, cognizance is given only to the results of the final series of tests, where end slots were eliminated, leaving free movement at the ends of the flash-boards along vertical walls at right angles to the center line of the gate. Notwithstanding turbulence, a possible slight degree of vacuum under the nappe, and varying rates of application of pressure, the steel pipe collapsed within sufficiently narrow limits to justify confidence that its behavior can be depended upon. The characteristic of uniform bending up to a critical yield angle from the vertical of from 20° to 30°, followed by a rapid collapse thereafter, is favorable for automatic flash-board control. Results based on

this series are analyzed, together with those of the mechanical tests, under the heading "Results of Experiments."

Mechanical Tests

Duplicate specimens 36 in. long were cut from the pipes used for the hydraulic tests and were used for mechanical tests. The specimens varied from $\frac{3}{4}$ in. to 3 in. in size. One specimen of wrought-iron pipe was tested also. The internal diameters were computed in the same manner as for the hydraulic tests; that is, from the weight and external dimensions.

The tests were made in a hydraulic testing machine of 100 000-lb capacity. As shown in Fig. 9, each pipe was loaded as a cantilever beam. The assembly

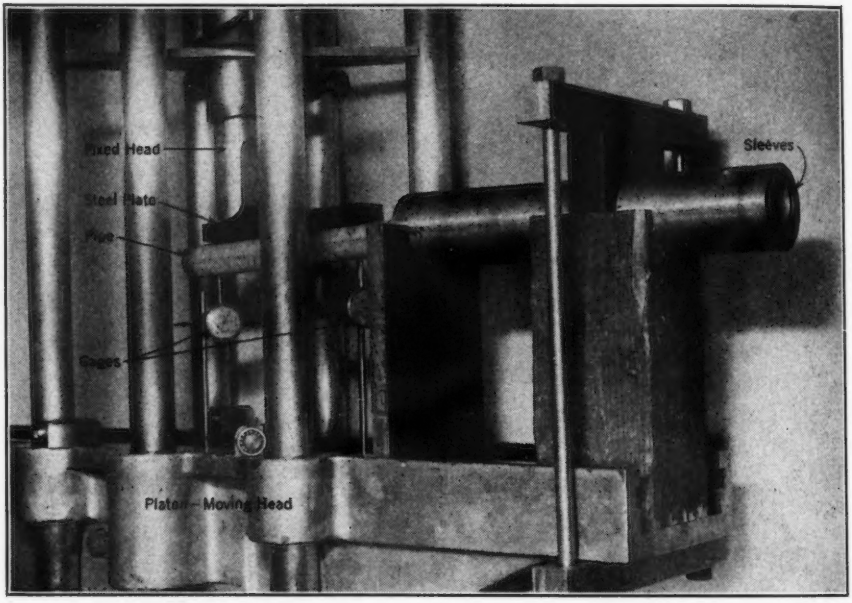

FIG. 9.—MECHANICAL TEST NO. 7, 1¼-INCH PIPE—PLATEN, GAGES, AND SLEEVES

of pipe sleeves used in the hydraulic tests was used also in the mechanical tests to simulate the same conditions of support. The load was applied to the pipe 10 in. from the support through a steel plate corresponding to the actual flash-board loading. During each test the load was applied in increments of approximately 200 lb, and the deflection of the end of the pipe for each load was computed from the readings of two dial gages. After the yield point of the pipe was passed and the pipe deflected excessively, the dial gages were removed, and the vertical distance from the pipe to the platen of the machine was measured until the maximum load was reached.

RESULTS OF EXPERIMENTS

General.—In arriving at the principal objective—the modulus of rupture and design of flash-board supports—the data from the tests were analyzed in the

following order:

(1) The actual bending moment on the pipe and the corresponding stress were computed as accurately as possible throughout the range of the test;

(2) Values of the modulus of rupture and the yield point for each pipe were determined for both the hydraulic and the mechanical tests;

(3) Values of the modulus of rupture determined for the hydraulic tests in Item (2) were used for the computation of the bending moment at which the pipe should fail; then, on this basis, formulas of various degrees of precision were set up for determining the corresponding water head;

(4) Simplified formulas for use in practical design were developed (moduli of rupture for use with these formulas were obtained by application of the formulas to the laboratory test heads);

(5) Using the values of modulus of rupture from Item (4) a simplified formula for standard sizes of pipe was set up for the prediction of the water head at failure;

(6) The laboratory set-up was assumed to represent a field installation. (The various design formulas, together with their appropriate value of modulus of rupture, were applied thereto and the resulting design head for failure was determined. These values were compared with the observed heads and the relative precision of the formulas determined);

(7) The simple method of design from Item (5) was applied to field structures and check tests were obtained; and,

(8) The original test data referring to Items (1), (2), and (4) are summarized in Table 2. Values in Column (14) are found by Equations (19*b*) or (20) (presented subsequently), divided by the section modulus for standard steel pipe given in Column (5), Table 3. The moment formulas in Items (3) and (4) are derived and discussed in the section on "Moment Formulas for Design." The heads computed for Item (6) are summarized in Table 3. The application of Items (5) and (7) is discussed in the section on "Design of Pipe Supports." Finally, suitable values of moduli of rupture and bending moment formulas are recommended in the "Conclusions." A summary of the bending moment formulas used in the computation of heads in Table 3 is presented in Table 4, the formulas being derived subsequently herein.

Notation.—The letter symbols in this paper are defined where they first appear and are assembled for convenience of reference in Appendix II. The term "kips," when used in this paper, denotes "kilo-pounds"; that is, thousands of pounds.

Computation of Stress in Pipe.—The cross-section of each pipe was assumed to be circular, and its section modulus was computed according to the standard formula:

$$S = \frac{\pi\,(D_e^4 - D_i^4)}{32\,D_e} \qquad (1)$$

in which: S = section modulus (Column (5), Table 2); D_e = external diameter of pipe; and D_i = internal diameter of pipe. The stress in the pipe was then

TABLE 2(*a*).—Hydraulic Tests; Final Series

Test *	External diameter, D_e, in inches	Wall Thickness: Thickness, t, in inches	Wall Thickness: Ratio, $\frac{t}{D_e}$	Section modulus, S, in inches³	Feet: Height of flash-board, h	Feet: Head, H, on crest of dam at failure	Deflection, θ, at failure, in degrees§	Maximum Moment, M, in Inch-Kips: Normal	Maximum Moment, M, in Inch-Kips: Due to weight of flash-board	Maximum Moment, M, in Inch-Kips: Total	Stress, f, in Pipe, in Kips per Square Inch: Yield point	Stress, f, in Pipe, in Kips per Square Inch: Modulus of Rupture: Normal moment¶	Stress, f, in Pipe, in Kips per Square Inch: Modulus of Rupture: Simplified
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Three-Quarter-Inch Pipe													
....	....	....	....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....	....	....	....
One-Inch Pipe													
26†	1.316	0.138	0.105	0.136	2.75	2.12	20	9.9	1.0	10.9	55.0	80.2	71.0
30‡	1.314	0.136	0.104	0.135	1.86	2.25	33	11.6	0.6	12.2	55.0	90.4	78.0
31‡	1.318	0.132	0.100	0.133	1.86	2.30	33	12.2	0.6	12.8	55.0	96.2	82.0
Av.	1.316	0.135	0.103	0.135	....	....	29	....	....	....	55.0	88.7	77.0
One-and-One-Quarter-Inch Pipe													
16†	1.663	0.136	0.082	0.232	3.83	2.54	20	17.8	2.0	19.8	50.0	85.4	70.0
....	....	....	....	....	....	....	....	....	....	....	....	....	....
Av.	....	....	....	....	....	....	....	....	....	....	....	....	....
One-and-One-Half-Inch Pipe													
17†	1.900	0.145	0.076	0.326	3.83	2.81	20	24.0	2.0	26.0	50.0	79.8	67.5
25†	1.900	0.141	0.074	0.318	3.83	2.83	20	24.4	2.0	26.4	55.0	84.0	69.0
27‡	1.901	0.150	0.079	0.336	2.31	3.00	21	24.2	0.6	24.8	50.0	73.8	71.5
28‡	1.901	0.150	0.079	0.334	2.31	3.15	22	26.9	0.6	27.5	....	82.4	78.5
29‡	1.899	0.156	0.082	0.342	2.31	3.20	22	27.6	0.6	28.2	....	82.4	81.0
Av.	1.900	0.148	0.078	0.332	....	....	21	....	....	....	52.0	81.3	73.5
Two-Inch Pipe													
18†	2.368	0.154	0.065	0.554	3.83	3.44	18	43.7	1.9	45.6	50.0	82.3	73.0
32‡	2.384	0.170	0.071	0.616	2.50	4.15	21	48.1	0.5	48.6	....	78.9	83.5
33‡	2.372	0.150	0.063	0.550	2.50	4.15	(33) ||	52.1	1.0	53.1	....	96.6 ||	83.5
(24)†	(2.372)	....	....	(0.604)	(3.83)	(3.24)	(18)	(36.2)	(1.8)	(38.0)	(35.0)	(62.9)	....
Av.	2.374	0.158	0.066	0.573	....	....	20	....	....	....	50.0	80.0	80.0
Two-and-One-Half-Inch Pipe													
19‡	2.863	0.206	0.072	1.068	3.83	4.16	17	74.0	1.7	76.0	40.0	71.2	66.5
22†	2.878	0.208	0.072	1.090	4.53	4.02	22	73.5	3.0	76.5	40.0	70.2	61.0
23†	2.866	0.214	0.075	1.103	4.53	4.20	18	79.9	2.5	82.4	....	74.6	70.0
Av.	2.870	0.210	0.073	1.087	....	....	19	....	....	....	40.0	72.0	65.8
Three-Inch Pipe													
20†	3.508	0.226	0.064	1.800	5.17	4.93	14	126.0	2.3	128.3	45.0	71.2	69.0
21†	3.504	0.216	0.062	1.730	5.17	4.84	14	119.0	2.3	121.5	45.0	70.2	65.0
Av.	3.506	0.221	0.063	1.765	....	....	14	....	....	....	45.0	70.7	67.0
General Average, Steel Pipe				....	....	....	21	....	....	....	49.2	79.6	73.0

* All specimens are steel except Tests 24 and 10*a*, which were wrought iron.
† Non-overflow tests.
‡ Overflow tests.
|| Not used in average; deflection excessive.
§ At maximum moment (modulus of rupture).
¶ Column (11) divided by Column (5).

TABLE 2(*b*).—MECHANICAL TESTS

Test*	External diameter, D_e, in inches	Wall Thickness: Thickness, t, in inches	Wall Thickness: Ratio, $\frac{t}{D_e}$	Section modulus, S, in inches³	Deflection, θ, at failure, in degrees§	Stress, f, in Pipe, in Kips per Square Inch: Yield point	Stress, f, in Pipe, in Kips per Square Inch: Modulus of rupture; normal moment
(1)	(2)	(3)	(4)	(5)	(8)	(12)	(13)
Three-Quarter-Inch Pipe							
1*a*	1.057	0.111	0.105	0.071	28	53.0	84.8
2*a*	1.047	0.118	0.113	0.071	32	53.0	86.8
Av.	1.052	0.115	0.109	0.071	30	53.0	85.8
One-Inch Pipe							
3*a*	1.314	0.138	0.105	0.135	23	52.0	79.8
4*a*	1.320	0.156	0.118	0.145	20	45.0	68.0
....	....	....	....	....	....	....	....
Av.	1.317	0.147	0.112	0.140	22	48.5	73.9
One-and-One-Quarter-Inch Pipe							
5*a*	1.659	0.137	0.083	0.229	23	45.0	68.6
6*a*	1.659	0.135	0.081	0.228	23	45.0	70.0
Av.	1.660	0.136	0.082	0.228	23	45.0	69.0
One-and-One-Half-Inch Pipe							
7*a*	1.894	0.149	0.079	0.329	23	45.0	70.7
8*a*	1.902	0.146	0.077	0.329	20	45.0	73.0
....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....
Av.	1.900	0.147	0.078	0.329	22	45.0	71.8
Two-Inch Pipe							
(10*a*)	(2.373)	(0.177)	....	(0.624)	(18)	(30.0)	(60.6)
9*a*	2.372	0.158	0.067	0.561	20	45.0	80.4
11*a*	2.368	0.177	0.075	0.607	19	30.0	59.0
Av.	2.370	0.168	0.071	0.584	20	37.5	69.7
....	....	....	....	....	....	....	....
Two-and-One-Half-Inch Pipe							
12*a*	2.867	0.216	0.075	1.102	21	43.0	74.5
13*a*	2.868	0.218	0.076	1.111	22	43.0	74.1
....	....	....	....	....	....	....	....
Av.	2.868	0.217	0.076	1.106	22	43.0	74.3
Three-Inch Pipe							
14*a*	3.505	0.231	0.066	1.804	17	50.0	75.0
15*a*	3.506	0.231	0.066	1.817	17	50.0	72.3
Av.	3.506	0.231	0.066	1.810	17	50.0	73.6
....	....	....	....	....	22	46.0	74.0

computed from the usual formula

$$f = \frac{M}{S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which M = bending moment in pipe; and, f = stress at the outer fiber.

Computation of Bending Moment.—Thick planks were used at the bottom of all of the flash-boards, and variations in height were obtained by changing the thin section at the top. Accordingly, the exact relation between the depth of the water and the bending moment produced by the water pressure on this irregularly-shaped flash-board could not be expressed readily in general terms;

TABLE 3.—COMPARISON OF PERFORMANCE; LABORATORY TESTS

Size of pipe, in inches	External diameter, D_e, in inches	Wall Thickness		Section modulus, S, in inches³	Adopted Modulus of Rupture, f, in Pounds per Square Inch	
		Nominal thickness, t, in inches	Ratio, $\frac{t}{D_e}$		Normal moment formula	Simplified horizontal moment formula
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(*a*) Standard Pipe Data for Design						
0.75	1.050	0.113	0.108	0.070	88 000	80 000
1.00	1.315	0.133	0.101	0.135	88 000	77 000
1.25	1.660	0.140	0.084	0.235	86 000	73 000
1.50	1.900	0.145	0.076	0.330	82 000	72 000
2.00	2.375	0.154	0.065	0.560	82 000	70 000
2.50	2.875	0.203	0.070	1.070	73 000	70 000
3.00	3.500	0.216	0.062	1.750	73 000	70 000
Average	....	....	....	....	....	....

TABLE 3.—(*Continued*)

Size of pipe, in inches	Height of flash-board, h, in feet	Designed Head for Failure, H, in Feet				Test head, at failure, in feet	Variation of Design Head from Test Head (Percentages)				Designed head, H, in feet*	Percentage variation of designed head (Column (18)) from test head
		Normal Moment ($\theta = 20°$); Flash-board Moment		Horizontal moment	Simplified horizontal moment formula		Normal Moment ($\theta = 20°$); Flash-board Moment		Horizontal moment	Simplified horizontal moment formula		
		Included	Neglected				Included	Neglected				
(1)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(*b*) Overflow Tests												
0.75	....	no tests	....	....	....	....	....	....	....	....	....	....
1.00	1.86	2.33	2.30	2.38	2.24	2.28	+2	+1	+4	−2	1.83	−20
1.25	....	no tests	....	....	....	....	....	....	....	....	....	....
1.50	2.31	3.16	3.12	3.22	3.02	3.12	+1	0	+3	−3	2.48	−20
2.00	2.50	4.05	4.02	4.11	3.75	4.15	−2	−3	−1	−10	3.03	−27
2.50	3.83	4.19	4.17	4.32	4.24	4.16	+1	0	+4	+2	3.66	−12
3.00	....	no tests	....	....	....	....	....	....	....	....	....	....
Average	....	....	....	....	....	....	+1	−1	+3	−3	....	−20

TABLE 3.—(*Continued*)

Size of pipe, in inches	Height of flash-board, h, in feet	Designed Head for Failure, H, in Feet: Normal Moment ($\theta = 20°$); Flash-board Moment: Included	Designed Head for Failure, H, in Feet: Normal Moment ($\theta = 20°$); Flash-board Moment: Neglected	Designed Head for Failure, H, in Feet: Horizontal moment	Designed Head for Failure, H, in Feet: Simplified horizontal moment formula	Test head, at failure, in feet	Variation of Design Head from Test Head (Percentages): Normal Moment ($\theta = 20°$); Flash-board Moment: Included	Variation of Design Head from Test Head (Percentages): Normal Moment ($\theta = 20°$); Flash-board Moment: Neglected	Variation of Design Head from Test Head (Percentages): Horizontal moment	Variation of Design Head from Test Head (Percentages): Simplified horizontal moment formula	Designed head, H, in feet*	Percentage variation of designed head (Column (30)) from test head
(1)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
(*c*) Non-Overflow Tests												
0.75	....	no tests	....	....	....	....	....	....	....	....	....	....
1.00	2.75	2.22	2.18	2.29	2.18	2.12	+5	+3	+8	+3	1.84	−13
1.25	3.83	2.61	2.61	2.72	2.58	2.54	+3	+3	+7	+2	2.21	−13
1.50	3.83	2.89	2.87	3.00	2.87	2.82	+3	+2	+6	+2	2.47	−12
2.00	3.83	3.46	3.43	3.58	3.39	3.44	−1	0	+4	−1	2.95	−14
2.50	4.53	4.12	4.10	4.27	4.21	4.11	0	0	+4	+2	3.66	−11
3.00	5.17	4.86	4.82	5.03	4.96	4.88	0	−1	+3	+2	4.31	−12
Average	....	....	....	....	....	....	+2	+1	+5	+2	....	−12

* For a (yield point) stress of 46 kips per sq in.

TABLE 4.—Summary of Bending Moment Formulas

Case	Formula	Equation No.	Computed head given in Table 3 in Column No.:
(*a*) Overflow Conditions			
1	$M = M_B{}^* + M_N = M_B + w\,L\,h^2\left(\frac{H}{2} - \frac{h\cos\theta}{3} - \frac{t\sin\theta}{2}\right)$	(4*b*) and (8)	(9)
2	$M = M_N{}' = w\,L\,h^2\left(\frac{H}{2} - \frac{h\cos\theta}{3}\right)$	(9)	(10)
3	$M = M_H = w\,L\,h^2\left(\frac{H}{2} - \frac{h}{3}\right)$	(10)	(11)
3(*a*)	$M = 375\,L\,h^2\left(H_1 + \frac{h}{3}\right)$	(19*b*)	(12)
(*b*) Non-Overflow Conditions			
4	$M = M_B + M_N = M_B{}^* + w\,L\left(\frac{H^3 - 3\,H^2\,t\sin\theta}{6\cos^2\theta}\right)$	(4*b*) and (15)	(21)
5	$M = M_N{}' = w\,L\left(\frac{H^3}{6\cos^2\theta}\right)$	(16)	(22)
6	$M = H_H = \frac{w\,L\,H^3}{6}$	(17)	(23)
6(*a*)	$M = 125\,L\,H^3$	(20)	(24)

* Computed separately.

so it was computed individually for each case. Later, simplifications were made in the assumed loadings, and general expressions for the moment were derived on that basis. A diagram similar to Fig. 10, showing all of the condi-

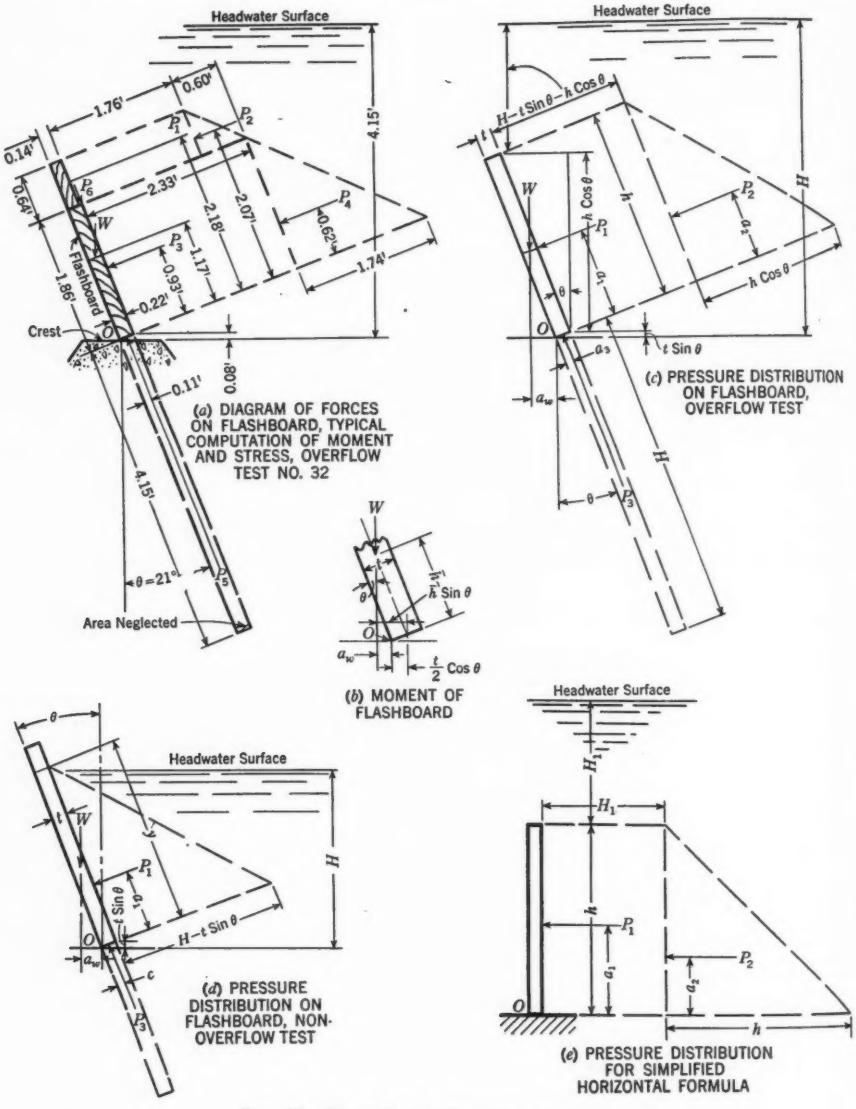

FIG. 10.—FLASH-BOARD PRESSURE DIAGRAMS

tions at failure, was made for each test, and the hydraulic-pressure distribution was determined for all parts of the flash-board, assuming static conditions. The effective bending moment, M, on the pipe was computed by taking the moments

of all the forces around the base of the board, O. The weight of the flash-board was appreciable in most of the tests, and hence its moment was added to that due to the water pressure. The moments are given in Columns (9) to (11), Table 2. These moments were computed individually for the exact conditions of the test and not from the simplified moment formulas, as was done for Column (14). The stress in the pipe was computed from the moment by the formula

$$M = f\,S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

An example of the computation of bending moment and pipe stress is given in Appendix III for the conditions shown in Fig. 10(a).

The assumption of static conditions obviously is correct as long as there is no water flowing over the flash-boards, and it is nearly correct, even when there is overflow, as long as both the deflection of the flash-board and the depth of overflowing water are small. No pressure measurements were made on the flash-boards, but unpublished tests made by the Pennsylvania Water and Power Company of Baltimore have shown that, as the depth of overflow, or the deflection, was increased, and the water moved up along the face of the board at an appreciable velocity, the actual pressure on the board was less than that due to the static conditions assumed. For the non-overflow tests, however, the assumption of static conditions was practically correct.

Stress-Deflection Curves.—The stresses in the pipe for each of the readings made before failure were computed by the same procedure, and stress-deflection curves were plotted for each test. Typical curves are shown in Fig. 11. The

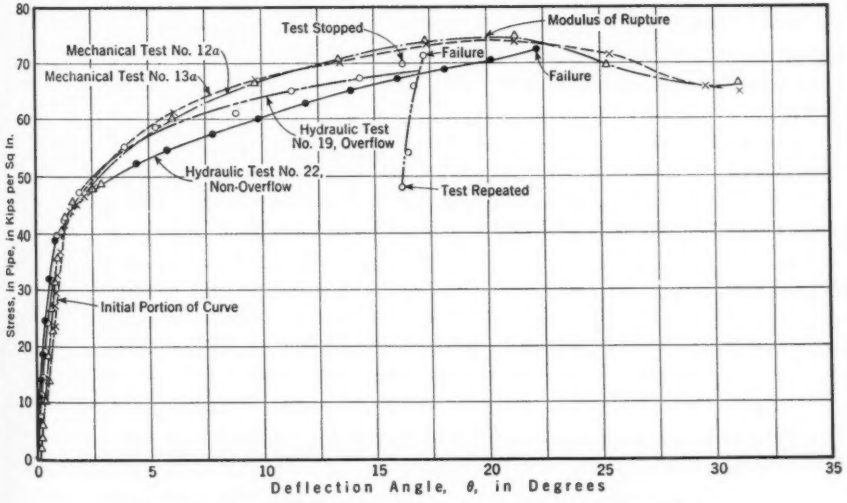

FIG. 11.—TYPICAL STRESS-DEFLECTION CURVES FOR A 2½-INCH PIPE

yield point (18) of the pipe for these tests was taken as the approximate stress at which the stress-deflection curve breaks away from its initial straight section. It was desired to determine the stress at which the pipes first began to bend

noticeably in these tests, since it was thought that they might possibly fail at the head causing this stress. When the deflection increases there is a considerable increase in the moment of the hydraulic load, even if the head remains constant. For this feature of the investigation, the term "yield point" seemed better adapted than other terms considered, although it does not accord with the standard definition. These values were determined from the curves and are given in Column (12), Table 2.

The stress computed from the maximum moment was called the "modulus of rupture," and these values are given in Columns (13) and (14), Table 2. (The term "modulus of rupture" has been used to indicate the maximum computed flexural strength, based on the original section of the pipe, in accordance with engineering practice.)

The stress-deflection curves for $2\frac{1}{2}$-in. pipe, shown in Fig. 11, were selected as typical of both the hydraulic and mechanical tests. The curves are seen to be generally similar in shape throughout their range but to differ in the value of the modulus of rupture. It may be observed in Fig. 11 that, as the headwater was lowered and again raised for Test No. 19, the corresponding stress-deflection curve tends to be parallel to its initial direction. Furthermore, as would be expected, the curve comes back to its original position before going on to the point at which the pipe failed.

The beginning of the stress-deflection curve, within the limits of error of the test, was observed to be a straight line and served to determine the approximate yield point as already described at about 46 kips per sq in. The deflection of the pipe over the elastic range of the test was very small—that is, less than 2° as compared to that at its subsequent failure at 20° to 30° deflection. Extensive studies of the properties of the pipe within this range of loading failed to yield any results pertinent to the main objective (design of flash-board supports); consequently no further consideration was given to conditions within the elastic range.

The deflection at modulus of rupture was plotted against the corresponding size of pipe for both the hydraulic and mechanical tests, using the data given in Table 2. The average values for the hydraulic tests showed a fairly consistent agreement with the mechanical tests. In both types of tests the deflection at failure is less for the larger pipes than for the smaller ones. A probable explanation of the latter fact is given subsequently herein in connection with the relation of modulus of rupture to pipe size. For the hydraulic tests the deflection ranged from 15° to 30° and the average was taken as 20° for later use in connection with the design of flash-boards.

Yield Point Stress.—The stress at the yield point for all of the pipes tested was found to range from 40 to 55 kips per sq in. averaging 49 kips per sq in. Beyond this stress permanent bending occurred. (It was found later, in field tests, that a value of 46 kips per sq in., computed by the horizontal formula, was more nearly in accordance with the facts; some tests gave as low as 40 kips per sq in.) However, failure did not occur until the pipe had traversed the large deflection angle of 15° to 30°, the angle of failure being consistent for pipes of the same size. Thus, for an actual installation, if the pipes had been

bent only slightly by a small flood, they still could be depended upon to fail at the desired head caused by a subsequent larger flood.

The wide spread between the yield point and the modulus of rupture of pipe supports is particularly significant. Instead of beginning decisively to fail at the yield point, the pipe did not actually fail until the stress value approached the modulus of rupture, although it was loaded beyond the yield point for a considerable period.

From these characteristics it is evident that in designing flash-board supports the diameter and spacing of the pipe should be such that the yield point, and resulting permanent bending of the pipe, will not be reached too frequently. The natural tendency is to build flash-boards too close to the maximum permissible water level in order to gain as much head and storage as possible. Then a small flood will bend the pipe, making the gate unsightly, or collapse it, requiring the pipe to be replaced. On the other hand, if the pins are designed for an initial stress of 30 to 50 kips per sq in., as well as for the ultimate modulus of rupture required, premature deformation due to debris, waves, or high water can be largely avoided.

The deviations of the heads at which the pipe should begin to bend, from the heads at which failure actually occurred, are shown in Columns (19) and (31), Table 3. These columns show, furthermore, that if the yield point is used in place of the modulus of rupture to compute the heads at which the pipes should fail, this will lead to values distinctly lower than the actual heads at failure. This comparison was made partly because of the belief (apparently held previously, but now shown not to be the case) that once the yield point of the pipes had been reached, the installation would go on to failure without any further increase in head.

Modulus of Rupture-Pipe Size.—In order to determine the proper modulus of rupture for the design of pipe supports, the relationship of modulus of rupture to pipe size was studied. The values of modulus of rupture given in Columns (13) and (14), Table 2, for both the hydraulic and mechanical tests, were plotted against the corresponding pipe size given in the sub-captions of the table (see Fig. 12). They range from 96 to 61 kips per sq in., and the values of the modulus of rupture, obtained from the hydraulic tests were approximately 20% greater for the smallest pipes than for the largest pipes. The values for the mechanical tests show a similar relationship between extreme sizes, although it is not so marked for the intermediate sizes. This relationship is due mainly to the greater relative wall thickness of the smaller pipes, which is indicated by comparing the values in Column (4), Table 2, with the standard pipe size. It is an accepted fact (19) that, when any structural member is stressed in bending beyond the elastic range, the stress distribution becomes non-linear, and the extreme-fiber stress computed on the assumption of a linear distribution exceeds the actual value. The greater the concentration of the material at the neutral axis, the greater this excess will be. Thus, although the actual stress occurring in the extreme fiber of two pipes may be the same, yet the computed value would be greater for the thicker-walled pipe. The larger pipes, with the relatively thinner walls, fail at smaller deflection angles and yield lower moduli of rupture.

Comparison of Hydraulic and Mechanical Tests.—For the mechanical tests the moments and resulting stresses were computed directly from the measured loads. The yield point for each pipe was determined from stress-deflection curves in the same manner as for the hydraulic tests and the results are given in Column (12), Table 2. Values of modulus of rupture determined from the maximum moment are given in Columns (13) and (14).

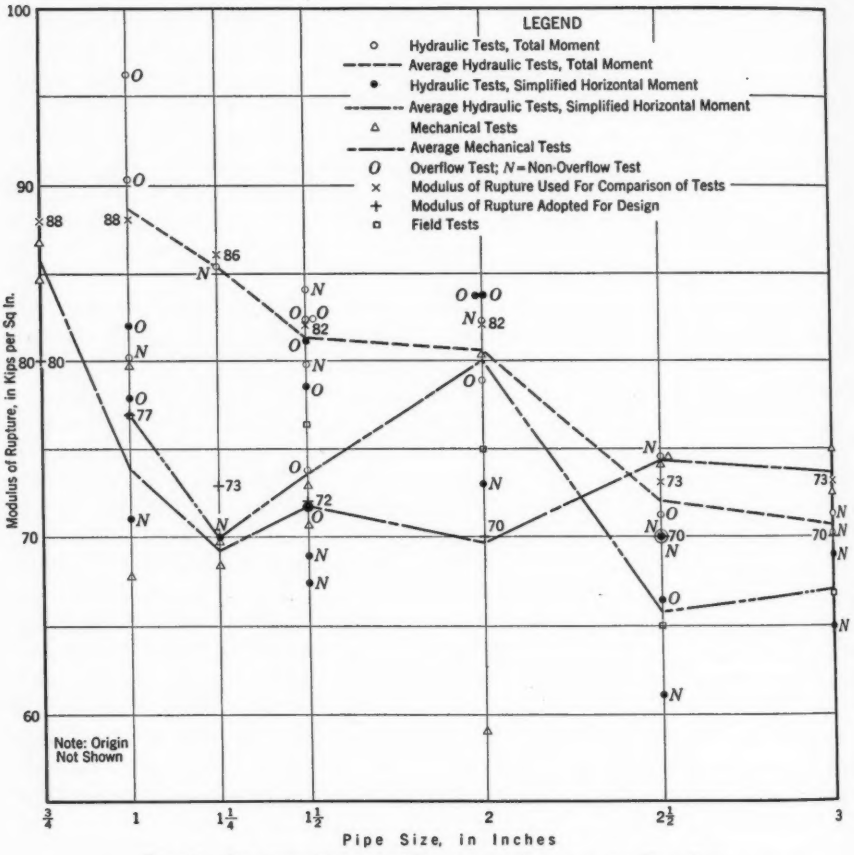

FIG. 12.—RELATION BETWEEN MODULUS OF RUPTURE AND PIPE SIZE

It is believed that the conditions were more nearly determinate, and more nearly in accordance with the assumptions made, in the tests of the 2½-in. and 3-in. pipes than for the smaller pipes. The results obtained in the mechanical tests agree quite well with those obtained in the hydraulic tests for the two largest sizes of pipe tested.

An exhaustive analysis of the discrepancies between the results of the mechanical and the hydraulic tests for pipe diameters of 2 in. and smaller was made (15*a*), but no obvious explanation of the lack of agreement was found. It was concluded that the discrepancy between the hydraulic and mechanical

tests is a function of any or all of a number of minor factors, none of which can be evaluated with any certainty. These include: (*a*) Uncertainty as to the portion of the water load that was actually transmitted to the pipes; (*b*) possible uncertainties as to the exact values of the moments applied in the mechanical tests, due to the large deflection at failure; (*c*) uncertainties as to the exact shape and condition of the pipe at the support; and (*d*) insufficient number of pipes tested. The values of modulus of rupture determined by the hydraulic and mechanical tests for the larger pipes are reasonable for the mild steel used in the pipes.

Average values, in round numbers, of the modulus of rupture obtained in the hydraulic tests for each size of pipe were determined for use in design and they are shown in Fig. 12.

Moment Formulas for Design

As indicated under Item (4) (see "Results of Experiments: General") simpler bending moment formulas were desired for practical design. Consequently three different simplifying assumptions were made, successively, and the resulting formulas derived as indicated subsequently. The resisting moments will be developed following the development of the bending moments.

Assumptions Made.—The formulas for the bending moment on the pipe at the dam crest are derived herein—first for overflow conditions and then for non-overflow conditions. In each case conditions are assumed in the following order: (*a*) The flash-boards were assumed to be of uniform thickness (Cases 1 and 4); (*b*) the weight and thickness of the flash-board were neglected (Cases 2 and 5); and (*c*), the flash-board in Item (*b*) was assumed to be swung back to the vertical (Cases 3 and 6). Static conditions were assumed for both the overflow and non-overflow derivations. The distribution of the pressures on the flash-board due to head-water for Case 1 (overflow) and Case 4 (non-overflow) is shown in Figs. 10(*b*) and 10(*c*), respectively.

Overflow Tests.—Taking moments about Point *O*, Fig. 10(*b*):

$$M = W\,a_w = W\left(\bar{h}\sin\theta - \frac{t}{2}\cos\theta\right) \ldots\ldots\ldots\ldots\ldots (4a)$$

and, since $W = \gamma\,L\,h\,t$ and $\bar{h} = \frac{h}{2}$:

$$M_B = \gamma\,L\left(\frac{h^2\,t}{2}\sin\theta - \frac{h\,t^2}{2}\cos\theta\right) \ldots\ldots\ldots\ldots\ldots (4b)$$

in which: W is the weight of the flash-board; $\bar{h}$ is the distance from the base of the flash-board to its center of gravity; a_w is the lever arm of the weight W; γ is the specific weight of the timber used for the flash-board (36 lb per cu ft); L is the length of flash-board along the crest supported by one pipe (average spacing of the supporting pipes); and, M_B is the moment due to the weight of the flash-board. The flash-board moment was appreciable only at deflection angles greater than 5 degrees.

Case 1. Normal Moment.—The flash-board is assumed to be of constant thickness t. The trapezoidal pressure diagram is divided into a rectangle and a triangle for convenience, as shown in Fig. 10(*c*). Consequently, the moment of the water pressure normal to the flash-board is:

$$M_N = P_1 a_1 + P_2 a_2 + P_3 a_3 \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which $a_1 = \frac{h}{2}$; $a_2 = \frac{h}{3}$; $a_3 = \frac{t}{2}$; and,

$$P_1 = w\,L\,h\,(H - t\sin\theta - h\cos\theta) \ldots\ldots\ldots\ldots\ldots (6a)$$

$$P_2 = \frac{w\,L\,h}{2}\,(h\cos\theta) \ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

$$P_3 = w\,L\,t\,H \left(\text{neglecting } \frac{t}{2}\sin\theta \right) \ldots\ldots\ldots\ldots\ldots (6c)$$

in which w is the specific weight of water (62.4 lb per cu ft). Hence:

$$M_N = w\,L\,h\,(H - t\sin\theta - h\cos\theta)\,\frac{h}{2} + \frac{w\,L\,h}{2}\,(h\cos\theta)\,\frac{h}{3} + w\,L\,t\,H\,\frac{t}{2} \ldots (7a)$$

or

$$M_N = w\,L\left(\frac{H\,h^2}{2} - \frac{t\,h^2\sin\theta}{2} - \frac{h^3}{2}\cos\theta + \frac{h^3}{6}\cos\theta + \frac{H\,t^2}{2}\right) \ldots (7b)$$

The term containing t^2, in Equation (7*b*), can be neglected in comparison with the other terms; and thus, the normal-moment formula becomes

$$M_N = w\,L\,h^2\left(\frac{H}{2} - \frac{h\cos\theta}{3} - \frac{t\sin\theta}{2}\right) \ldots\ldots\ldots\ldots\ldots (8)$$

Case 2. Normal Moment Neglecting Weight and Thickness of Flash-Board.—Making $t = 0$ in Equation (8), the normal moment, neglecting the weight and thickness of the flash-board, is:

$$M_N' = w\,L\,h^2\left(\frac{H}{2} - \frac{h\cos\theta}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Case 3. Normal Moment Considering Flash-Board Swung Back to the Vertical, and Neglecting Weight and Thickness of Flash-Board.—In Equation (9), make $\theta = 0$, and the normal moment (considering the flash-board to be swung back to the vertical, and neglecting the weight and thickness of the flash-board) becomes:

$$M_N'' = M_H = w\,L\,h^2\left(\frac{H}{2} - \frac{h}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

which is the horizontal moment formula.

Non-Overflow Tests.—The moment of the flash-board itself is the same as before (Equation (4*b*)).

Case 4. Normal Moment.—In Fig. 10(*d*) the moments are taken around Point O as in Equations (4):

$$M_N = P_1 a_1 + P_3 a_3 \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which $a_1 = \frac{y}{3}$; $a_3 = \frac{t}{2}$; y = slant height, wetted surface of flash-board; and,

$$P_1 = \frac{w\,L\,y\,(H - t\sin\theta)}{2} \quad \ldots\ldots (12a)$$

$$P_3 = w\,L\,t\,H \quad \ldots\ldots (12b)$$

$$M_N = w\,L\,y\,\frac{(H - t\sin\theta)}{2} \times \frac{y}{3} + w\,L\,t\,\frac{H\,t}{2} \quad \ldots\ldots (12c)$$

Since

$$y = \frac{H - t\sin\theta}{\cos\theta} \quad \ldots\ldots (13)$$

Therefore,

$$M_N = w\,L\left[\frac{(H - t\sin\theta)^3}{6\cos^2\theta} + \frac{t^2\,H}{2}\right] \quad \ldots\ldots (14)$$

In Equation (14), expand $(H - t\sin\theta)^3$ and neglect terms containing t^2 and t^3; and the normal-moment formula is found to be:

$$M_N = w\,L\left(\frac{H^3 - 3\,H^2\,t\sin\theta}{6\cos^2\theta}\right) \quad \ldots\ldots (15)$$

Case 5. Normal Moment Neglecting Weight and Thickness of Flash-Board.—If $t = 0$ in Equation (15), the normal moment, neglecting the weight and thickness of the flash-board, becomes:

$$M_N' = w\,L\left(\frac{H^3}{6\cos^2\theta}\right) \quad \ldots\ldots (16)$$

Case 6. Normal Moment Considering Flash-Board Swung Back to the Vertical, Neglecting Weight and Thickness.—Considering the flash-board swung back to the vertical and neglecting weight and thickness, $\theta = 0$ and $t = 0$, from which, by Equation (15), the horizontal-moment formula becomes:

$$M_N'' = M_H = \frac{w\,L\,H^3}{6} \quad \ldots\ldots (17)$$

Case 3(a) Simplified Horizontal Moment Formula.—Since it was found that compensating refinements, such as weight and thickness of flash-boards, angle of deflection, and increase of normal moment over horizontal moment, had no practical effect in the tests, the horizontal moment has been adopted as a working basis for design (see Item (7), heading "Results of Experiments: General"). The formulas were simplified for field and office use as follows (see Fig. 10(*e*)).

Overflow Tests.—In Equation (10), written in the form,

$$M_H = w\,L\left(\frac{H\,h^2}{2} - \frac{h^3}{3}\right) = M \quad \ldots\ldots (18)$$

substitute $w = 62.4$ lb per cu ft and $H_1 + h = H$, in which H_1 is the head of water, in feet, above the top of the flash-board. Then, the moment, in foot-

pounds, is

$$M = 62.4\, L \left(\frac{H_1\, h^2}{2} + \frac{h^3}{6} \right) \ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

or, in inch-pounds,

$$M = 375\, L\, h^2 \left(H_1 + \frac{h}{3} \right) \ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

Non-Overflow Tests.—From Equation (17), the moment, in inch-pounds, is

$$M = 125\, L\, H^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Comparison of Computed with Measured Heads at Failure.—As indicated under Items (5), (6), and (7) (heading, "Results of Experiments: General"), the next step in the investigation was to compare the heads at which flash-board installations should fail, as computed by the various bending moment formulas (Table 4), with the heads at which failure actually had been observed to occur on installations. The laboratory installations used in this investigation to determine the modulus of rupture obviously afforded a convenient opportunity to make such a comparison. The flash-boards were assumed to be supported by standard steel pipe having the dimensions and section moduli given in Table 3. The moduli of rupture used were selected from Fig. 12 and are given in Column (6) of Table 3. For each size of pipe used, the two quantities, S and f, determined the resisting moment to be used in design. No allowance was made for any partial vacuum under the sheet of water flowing over the flash-boards, since they were ventilated at each end.

It should be noted carefully that this is really a comparison of the relative accuracy with which various approximate formulas, nominal pipe sizes, and moduli of rupture will determine the head at failure of a flash-board installation, as compared to the actual head at which failure occurred. The discrepancies found might be somewhat larger when comparisons are made with other installations.

The data for standard pipe, the dimensions of the flash-boards, and spacing of pipe were substituted in the bending moment formulas (Table 4) and the head at which the flash-board should fail was determined (Table 3).

The heads at which the pipes should begin to bend were computed for the horizontal formula using a stress of 46 kips per sq in. to determine the bending moment. The resulting heads are given in Columns (18) and (30) of Table 3. The heads at which pipe should fail are given in Columns (12) and (24); and, the heads at which failure actually occurred are given in Columns (13) and (25).

The differences between the computed heads and the measured heads are given in Columns (14) to (17) and (26) to (29). The values obtained from the normal-moment formula, including the moment due to the flash-board (Cases 1 and 4), and similar values neglecting the flash-board moment (Cases 2 and 3), are seen to be very nearly the same for both the overflow and the non-overflow tests. The average difference ranges from 2% high to 1% low, with a maximum variation from + 5 to − 3 per cent.

The more approximate and convenient simplified horizontal moment formulas, neglecting both the flash-board moment and the angular deflection

(Cases 3*a* and 6*a*), yielded results within an average of 3% lower than the test heads for the overflow and 2% higher for non-overflow tests. For this formula, values of modulus of rupture developed in the following section and given in Column (7), Table 3, were used. It is seen that the more accurate and complicated formula for normal moment, allowing for both the weight and thickness of the flash-board, probably does not improve the results sufficiently to justify its use. Therefore, Equation (20) for non-overflow tests, and Equation (19*b*) for overflow tests, are recommended for use in the selection of pipe for flash-board supports.

Modulus of Rupture.—In order to determine suitable values of modulus of rupture for design with Equations (19) and (20), values of the test data from Table 2 were first substituted in the formulas and the modulus of rupture computed. They are given in Column (14), Table 2. In Fig. 12, they are seen to be in very close agreement with those of the mechanical tests. Average values of the modulus of rupture in round numbers, to be used in Equations (19) and (20) for flash-board design, were selected for each pipe size.

Design of Pipe Supports

The use of the derived formulas Equations (19*b*) and (20), in combination with the relation $M = f S$ for the design of the flash-board pipes, can be reduced to a very simple procedure. Since the variables involved are the height of the water above the spillway, height of flash-board, and span between supports, it is customary to designate two of these and solve for a third. Usually the size of pipe is to be determined. If so, and the water level is fixed, it may be necessary to modify the pipe spacing by trial to compensate for the gap between standard pipe sizes. On the other hand, if the pipe size and spacing are fixed, a definite solution for the head results.

Variable Modulus of Rupture.—The maximum bending moment, being a function of the known section modulus of the pipe (computed from standard dimensions) and of the experimentally-determined modulus of rupture, is determined by the latter. This characteristic, as opposed to the expected constant modulus of rupture for a given material, such as steel, constitutes an outstanding feature of the tests. It means that for accuracy, different values of modulus of rupture must be used in determining the maximum bending moment for different sizes of pipe since the modulus of rupture ranges from approximately 70 kips per sq in. for 3-in. pipe to 90 kips per sq in. for ¾-in. pipe (see Fig. 12). It is here that the test results are most valuable, inasmuch as the range in modulus of rupture is by far the greatest variable in the design. Therefore, for reasonable accuracy in the control of head-water elevations, it is preferable to use the tabulated modulus of rupture that corresponds most nearly to the selected pipe size, rather than to assume an average value for all sizes of pipe. As already indicated, if pipe sizes larger than 3 in. are to be used, it is presumed that they will first be checked by additional tests.

Height of Flash-Board.—It is obvious from Equations (19) and (20) that the ratio of the height of the flash-board to the total head of water is important. For the same pipe and spacing, the higher flash-boards fail earlier or under less

head of water than do lower boards. To express it another way, for a given height of water above a spillway crest, a high flash-board gate with only a few inches of water flowing over it will fall, whereas a lower flash-board (supported by the same pipe system), overtopped by several feet of water depth, will stand erect. This inherent characteristic must be considered in selecting the appropriate flash-board dimensions for a structure.

Hinges.—If a flash-board gate is hinged at the bottom, the hinges must be loosely fitted. Any binding at the hinge would prevent the flash-board either from swinging freely under water pressure or from transferring its load properly to the pipe support. In either case the load on the pipe would be reduced and it would fail at a higher head than designed.

Typical Design Procedure.—To illustrate design procedure assume a spillway channel 35 ft wide by 8 ft deep, the entire capacity (depth) of which is required in time of flood. It is desired to raise the normal lake level 3 ft (h) above the spillway-level (crest), limit the maximum flood height above this normal water level to 2 ft (H_1), and, at the same time, retain a 3-ft free-board between the maximum flood height and the top of the dam. The cost of widening this spillway channel, so that it would have the same flood capacity at a 5-ft depth, compared to this 8-ft depth, would be prohibitive. The procedure follows:

(1) Pipe spacing: The customary spacing for the supporting pipe for a gate of this height is from 3 to 5 ft since wider spacing requires a gate that is too ponderous to lift back into position after failure, and closer spacing multiplies pipe replacement units. Try eight pipe supports. Then the length supported by one pipe is $\frac{35}{8} = 4.4$. However, since the gate is sufficiently rigid to distribute the loading equally to all the pipe, they act as a whole and are best designed on a basis of total moment. To prevent "curling" at the ends, place a pipe 1 ft from each end of the gate and space the remaining six pipes symmetrically with respect to the center line of the channel.

(2) Determine pipe sizes: Use the overflow formula (Equation (19b)) for bending moment, with $L = 35$ ft. Then $M = 375 \times 35 \times 9\,(2 + \frac{3}{3}) = 354\,000$ in-lb. The section modulus $S = \frac{M}{f}$ with $f = 70\,000$ lb per sq in. (approximately) yields: $S = \frac{354\,000}{8 \times 70\,000} = 0.632$ which indicates pipe between 2-in. and $2\frac{1}{2}$-in. in diameter. By trial it is found that a $1\frac{1}{2}$-in. pipe at each end, with four 2-in. and two $2\frac{1}{2}$-in. pipes alternated symmetrically between, gives a resisting moment of 353 000 in-lb. This is within the working limits.

(3) Check of yield point: Substituting again the combined section moduli of these eight pipes in $M = Sf$ when $f = 46\,000$ lb per sq in. (yield point) yields: $M = 5\,040 \times 46\,000 = 232\,000$, which is equivalent to a head over the gate (Equation (19b)) of $H_1 = \frac{232\,000}{375 \times 35 \times 9} - \frac{3}{3} = 0.9$ ft. If it is probable that a head of 0.9 ft over the gate will occur frequently it would be better either to lower the gate height or use stronger pipe. This encroaches on the free-board during occasional flood peaks, but avoids frequent unsightly bending of the pipe.

Installation at Sherando Lake Tested

An automatic flash-board gate 35 ft long by 3.2 ft high, designed in accordance with the findings of the Hydraulic Laboratory tests, was installed at Sherando Lake, 10 miles south of Waynesboro, Va., in the George Washington National Forest in the fall of 1935 (see Fig. 1). The wooden gate is hinged at eight points and is sufficiently rigid so that the hydraulic load is apparently equal on all of the eight supporting pipes. These pipes are spaced on about 5-ft centers with end pipes about 1 ft from the channel side-walls. The average length of gate supported by one pipe is 4.4 ft. The hinges are placed behind the pipes and are very loosely fitted to avoid binding or taking part of the pipe load.

Floods collapsed the gate three times during 1936. First, on March 20, when the lake level reached a head of 2.42 ft (determined by an automatic gage) over the top of the flash-board, or within 0.12 ft of that specified, the gate opened gently downward with an accelerating speed as it fell into the surging tail-water which cushioned its fall. At first the gate leaned more and more out of plumb until the angle was about 20°; then all pipes collapsed and fell simultaneously.

Following this successful field test of 1-in. pipe, the pipe size was increased progressively, using first $1\frac{1}{2}$-in. pipe and then 2 in. with $1\frac{1}{4}$-in. end pipes. In each case the gate fell when the water surface was 0.1 to 0.3 ft below the designed height—that is, on the safe side. This procedure of first testing a pipe size smaller than the design calls for is recommended where the pipe cost is negligible. This helps to ascertain the effect of local conditions—such as wind, wave and current action, ice thrust, presence of logs and debris, vibration, and partial vacuum under the gate.

Other Installations.—One gate costing $500 saved a spillway cost of $15 000. This particular gate was 30 ft long and consisted of two superimposed vertical sections, the upper 2 ft being designed to fail under a 1.5-ft head and the lower $4\frac{1}{2}$-ft section failing under 3-ft head. Vacuum under the nappe caused this gate to fall at about 7% lower head than would be the case with ventilated overflow.

Checks made on the operation of a number of installed flash-board gates indicate adherence to within about 5% of the design water heads, and in all cases the failure of the gate has been on the safe side.

It has been observed that, in installations where a considerable degree of vacuum is formed beneath the nappe, the gate fails under a lower head-water than otherwise. This offers the alternative of using heavier pipe supports or of breaking the vacuum by vent pipes or by other expedients. However, like all other field test results obtained by following this design method, this vacuum effect tends to lower the maximum flood stage behind the dam.

Conclusions

The following conclusions have been amplified to include some minor, as well as the major, results of the experiments and field tests because of the dearth of published information on flash-board installations.

(1) Flash-boards with standard, steel-pipe supports can be designed to fall consistently within narrow limits under specified conditions.

(2) In the design of pipe supports for flash-boards, the simplified horizontal moment formula (Equations (19*b*)) for ventilated overflow conditions, and Equation (20) for non-overflow conditions, will give results within practical limits (approximately 3 per cent). Stock commercial steel pipe can be used, the section modulus being computed from internal and external diameters taken from lists of pipe standards as shown in Table 2. The moduli of rupture and resulting bending moments recommended for use in these formulas are given in Table 5. The stress at the yield point may be taken as 46 000 lb per sq in.

TABLE 5.—RECOMMENDED VALUES, MODULI OF RUPTURE AND RESULTING BENDING MOMENTS

Description	PIPE SIZES (NOMINAL DIAMETER) IN INCHES						
	¾	1	1¼	1½	2	2½	3
Modulus of rupture, in pounds per square inch	80 000	77 000	73 000	72 000	70 000	70 000	70 000
Bending Moment, in inch-pounds	5 600	10 400	17 150	23 800	39 200	74 900	122 500

(3) If greater accuracy is desired, the appropriate normal-moment formulas (Equations (4), (8) or (15)), may be used with the moduli of rupture shown in Column (6), Table 3.

(4) The modulus of rupture to be used in design may be determined experimentally from mechanical as well as hydraulic tests for pipes larger than 1 in. in diameter.

(5) The ratio of height of flash-board to total head of water should be such that ordinary floods can be passed without permanent bending of the pipe supports. In other words, the yield point of the pipe must be taken into consideration in design.

(6) Any unusual local conditions must be considered in the application of the formulas.

(7) The load on flash-board pipes may be reduced and increased successively without producing failure, as long as the maximum load at which failure should occur is not reached. When the maximum load (head) is reached, however, the pipe will assuredly collapse and fall completely.

(8) Uniform bending up to a critical angle with the vertical, of from 20° to 30°, usually precedes failure.

(9) Vacuum under the nappe causes vibration and collapse of the flash-board pipes at lower heads than designed. This is generally on the safe side.

(10) Because of loading conditions and relative wall thickness, the modulus of rupture used in designing steel pipes for flash-board supports is considerably greater than the minimum specified ultimate tensile strength of the metal in the pipe. This variation is greater for the smaller pipes.

(11) The formulas recommended for design (see Table 4) do not allow for the indeterminate binding effect of wedge, or "V"-jointed, flash-boards, or the

interlocking and wedging of "butt"-jointed flash-boards, in preventing failure at the desired elevation of the head-water.

(12) The use of solid steel or iron pins for flash-board supports seems to be not as dependable as the use of standard steel pipe.

Acknowledgments

The success of the work was due largely to co-operation between the U. S. Forest Service and the National Bureau of Standards.

All hydraulic tests were conducted in the Hydraulic Laboratory Section of the latter under the general supervision of H. N. Eaton, Chief of the Section. Mr. Eaton also carefully reviewed and edited the previous report as well as the manuscript of this paper. The apparatus was constructed by C. W. Elliot of the laboratory staff, and in making observations, other members of the staff, as well as men from the office of the Forest Service, participated. The mechanical tests were made in the Engineering Mechanics Section by A. H. Stang, and others, under the supervision of H. L. Whittemore, Chief of the Section. The work of these laboratories is supervised generally by H. L. Dryden under L. J. Briggs, Director of the National Bureau of Standards.

J. C. Dort, M. Am. Soc. C. E., proposed the laboratory experiments to check the design for the installation of flash-boards at Sherando Lake. F. L. Brown, Jun. Am. Soc. C. E., was assigned to the hydraulic laboratory during the tests and A. H. Ronka, Assoc. M. Am. Soc. C. E., assisted in the analysis of results.

Regional Forester, R. M. Evans, is Chief of Region 7, U. S. Forest Service. T. W. Norcross, M. Am. Soc. C. E., is Chief of the Division of Engineering, U. S. Forest Service, and F. A. Silcox is Chief of the Forest Service.

The Pennsylvania Water and Power Company of Baltimore loaned their voluminous test data freely and facilitated inspection of their flash-board installations.

APPENDIX I

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APPENDIX II

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[5] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

a = lever arm to a local force, P; a_1 = lever arm to P_1; a_2 = lever arm to P_2; a_3 = lever arm to P_3; and a_w = lever arm to the force W;

D = diameter of pipe; D_e = external diameter; and, D_i = interior diameter;

f = stress in the outer fiber of a pipe;

H = height of head-water above the crest of the dam at failure; H_1 = height of water surface above the top of the flash-board;

h = height of the flash-board above the base of the hinge; $\bar{h}$ = distance from the base of the flash-board to its center of gravity;

[5] A. S. A.—Z10a—1932.

L = length of flash-board, along the crest, that is supported by one pipe; average spacing of the supporting pipes;

M = maximum bending moment; M_B = the moment due to the weight of the flash-board; M_N = moment due to water pressure normal to the flash-board; the total moment acting on a supporting pipe is $M_N + M_B$; M_N' and M_N'' = the approximate moment due to water pressure on the flash-board; M_H = the moment due to water pressure on a flash-board assumed to be swung back to the vertical position—that is, the "horizontal moment";

P = concentrated loads; P_1, P_2, etc. are local forces;

S = section modulus;

t = average thickness of the flash-board;

W = weight of flash-board, in pounds per pipe;

w = specific weight of water (62.4 lb per cu ft);

y = slant height, wetted surface of flash-board;

γ = specific weight of the timber used for flash-boards;

θ = angular deflection of the flash-board from the vertical.

APPENDIX III

Computation of Moment and Stress

Overflow Test No. 32.—The pressure diagram for Test No. 32 is shown in Fig. 10 (a) and the pipe data are given in Table 2. The distribution of water pressure on the flash-boards is shown diagrammatically, and the force diagram thus obtained is divided into convenient areas for computing the moment. All dimensions are in feet. The summation of the moments of all the areas around the base at Point O represents the total water moment. The moment of W about Point O is the flash-board moment.

Referring to Fig. 10 (a): $H = 4.15$ ft; $W = 150$ lb; $\bar{h} = 1.17$ ft; $\theta = 21°$; $M = M_B + M_N$; and,

$$M_B = W(\bar{h} \sin \theta - 0.11 \cos \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Substituting appropriate values in Equation (21), $M_B = 540$ in-lb.

For normal moment take moments around Point O:

$$M_N = P_1(2.18) + P_2(2.07) + P_3(0.93) + P_4(0.62) + P_5(0.11) - P_6(0.18) \quad \ldots\ldots (22)$$

Again, substituting appropriate values in Equation (22), $M_N = +7.96\,(12\,w\,L)$ in-lb; and, neglecting P_6, $M_N = 48\,130$ in-lb. Then, $M = M_B + M_N = 500 + 48\,100 = 48\,600$ in-lb.

The stress in the pipe (modulus of rupture) $f = \dfrac{M}{S} = \dfrac{48\,600}{0.616} = 78\,900$ lb per sq in. as shown in Column (13), Table 2.

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PAPERS

TENSION TESTS OF LARGE RIVETED JOINTS

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Synopsis

An extensive series of tests on large riveted joints is described in this paper, including variations in length, type of splice, plate and rivet steel, pitch, and rivet pattern. The paper discloses the behavior of the various parts under load and provides information bearing on design practices. It was found that the usual assumption of equal distribution of load among rivets is satisfactory for design purposes, regardless of length of joint; and, that joints having as many rivets in the end row as in interior rows are as efficient as corresponding joints in which the net section is supposedly protected by using fewer rivets in the end row.

Introduction

The specifications commonly followed in the design of structural riveted connections are based largely on tests made at Cornell University, at Ithaca, N. Y., in 1904.[4] The joints tested at that time were relatively small, having a maximum of nine rivets in either end, and were made of softer plate, and rivet, steels than those now used even for ordinary bridges and buildings. Tests on joints fabricated with nickel-steel plates and rivets were conducted at the University of Illinois, at Urbana, Ill., in 1911;[5] but, again, there were no more than nine rivets in either end of the joint. Tests involving a larger number of rivets have been confined almost entirely to short joints of the type used in ship, and tank, construction.[6, 7] Experimental data on the behavior of

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[6] "An Investigation of the Behavior and of the Ultimate Strength of Riveted Joints Under Load," by E. L. Gayhart, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 34, 1926, p. 55.

[7] "Tests of Joints in Wide Plates," by Wilbur M. Wilson, M. Am. Soc. C. E., James Mather, and Charles O. Harris, Eng. Experiment Station, *Bulletin No. 239*, University of Illinois, 1931.

long joints are rare, although theoretical analysis indicates that increasing the number of transverse rivet rows in each end above, say, five results in lower efficiency.

In connection with the design and construction of the San Francisco-Oakland Bay Bridge, it was found possible, in 1934, to begin two groups of tests with the object of securing information on the behavior of long riveted joints containing carbon and alloy steels and on related problems. One group, consisting of fatigue tests of relatively small joints, was conducted at the University of Illinois.[8] The other group, comprising static tension tests of large joints, was conducted at the University of California; these tests are described herein.

The program of static tension tests was planned to secure information on the following points: (1) The strength of large joints containing as many as twenty-eight transverse rows of rivets; (2) The relative behavior of joints having rivets of carbon or manganese steel; (3) The relative behavior of joints having plates of carbon, silicon, or nickel steel; (4) The relative merit of butt and shingle splices; (5) The partition of load among the transverse rows of rivets; (6) The effective net section of riveted plates; (7) Slip phenomena in large joints; and, (8) The proper design relation between tensile stress in plates and shearing stress in rivets. Throughout the tests, effort was made to secure data as to the action of the joints within the working range.

The abbreviation "kip" used throughout this paper denotes "kilo-pounds"; that is, 1 000 lb.

The Action of Riveted Joints

As an aid to the interpretation of test results, it is desirable to review certain general concepts of the action of riveted joints. These views are derived in part from theoretical considerations and in part from the results of these tests.

If perfect fabrication could be attained, the following conditions would exist: (*a*) Plates absolutely flat; (*b*) faying surfaces in contact at time of riveting; (*c*) holes matched perfectly; (*d*) at time of driving, holes completely filled by rivets; and (*e*) after cooling, rivets concentric with holes, with annular space around each rivet. During cooling, owing to the effort of the rivets to contract longitudinally against the restraint of the plates, tension is developed in the shank of the rivets. Lateral contraction of the rivet shank is caused in part by thermal contraction and in part by the Poisson-ratio effect due to rivet tension. The pressure between the plates caused by rivet tension results in a frictional resistance of the plates against slipping.

The behavior, under load, of a perfectly fabricated, two-rivet joint such as that shown in Fig. 1(*a*) may be considered in four stages: In the first stage static friction prevents slip; in the second stage the load is greater than the static friction, and the joint slips until the rivets come into bearing; in the third stage, rivets and plates deform elastically so that the load-slip relation is linear; and, in the fourth stage, yielding of plates, rivets, or both, occurs until either plate fracture or complete shearing of the rivets results. This behavior

[8] "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson and Frank P. Thomas, Eng. Experiment Station, *Bulletin No. 302*, University of Illinois, 1938; see also "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson, *Civil Engineering*, Vol. 8, August, 1938, pp. 513–516.

is indicated by the diagrams of Fig. 1(*b*), which are reproduced from typical results of pilot tests on two-rivet joints tested at the University of Illinois[8] in connection with this investigation. The range of load over which Stage I extends is affected by the amount of friction between the plates. The friction depends on the tension in the rivets and the characteristics of the faying surfaces.

In Stage II, as slip progresses, the tilting of the rivets increases the pressure between the plates and hence increases the frictional resistance to movement. In lap joints, the bending due to eccentricity tends to cause the plates to peel

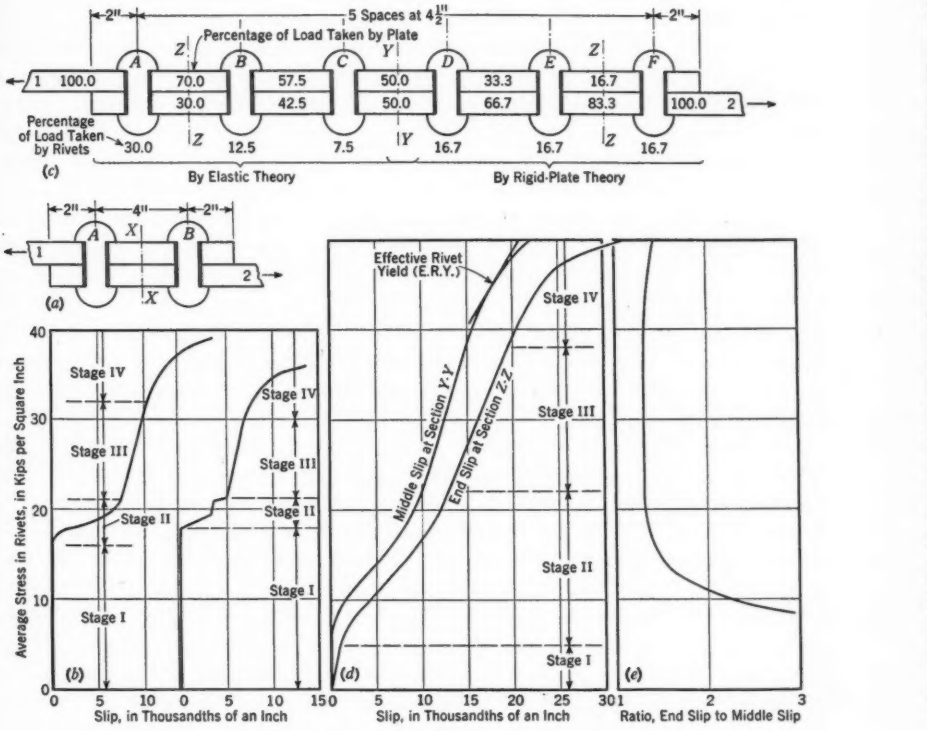


FIG. 1.—ACTION OF RIVETED JOINTS

apart at the ends and affects the amount of friction. These actions, together with the effect of non-uniform clearance around rivets, bending of the rivets, and distortion where contact is made between plates and rivets, extend Stage II over a range of load and cause the load-slip relation to take various forms.

Fig. 1(*c*) is a diagram of a joint with six rivets in line, showing the proportion of load transmitted by the rivets and the partition of load between plates, computed by the "elastic theory,"[9] which is based on the assumption that the rivets and plates deform elastically. Plate stresses in Fig. 1(*c*) are also com-

[9] "Work of Rivets in Riveted Joints," by A. Hrennikoff, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 437–489.

puted on the assumption of equal load partition among rivets, which may be called the "rigid plate" theory. It is seen that by the elastic theory the end rivets take a relatively high percentage of load.

In Stage I, load is transferred from one plate to the other by friction. Stage I ends when the load is sufficient to cause movement of the joint as a whole—that is, when slip begins at the middle of the joint. This is shown by Fig. 1(*d*) in which the load-slip diagrams are generalized from the results of several tests in this investigation.

Although it is conceivable that the plates are in close contact over their entire faying surfaces, it seems likely that, in actual joints, owing to quilting action, the plates develop effective frictional resistance only over relatively small annular areas around the rivets. These localized areas of contact may be thought of as spot welds; and, in Stage I the load may be thought of as transferred from one plate to another, not by actual shear in the rivets, but by these "spot welds." The results of tests show that, barring irregularities caused by imperfect fabrication, the stress distribution in the plates in Stage I is similar to that computed by the elastic theory.

Under either the elastic or the rigid-plate theory, at any section except at the middle of the joint, the stresses in the two plates differ. Owing to the difference in stresses, the plates tend to elongate by unequal increments, and therefore relative movements of the plates tend to occur at the ends, although slip of the joint as a whole has not occurred. As indicated by Fig. 1(*d*), slips measured near the end of a joint having several rows of rivets begin as soon as load is applied. Furthermore, the end slips in Stage I are practically proportional to load.

In Stage II, slip at any section of a joint increases at a greater rate than load; the action is similar to the yielding that occurs at higher loads. Frictional resistance to movement probably increases due to the plates being pressed closer together by the tilting of the rivets, and due to tooth action arising from roughness of the plates. The effect of this yield action in Stage II is to redistribute the load between the various rivets to values between those calculated by the elastic and the rigid-plate theories. As slip continues, the rivets come progressively into bearing against the plates, and the load-slip relation changes as indicated by a reversal in the direction of curvature of the slip diagrams. In lap joints having four to six rows of rivets, Stage II begins at average rivet stresses of perhaps 10 kips per sq in., or less, and ends at average rivet stresses of about 15 to 25 kips per sq in. Thus, for the average riveted joint, the working range lies largely within Stage II.

In Stage III, where elastic action takes place with all rivets in full bearing, the ratio of end slip to middle slip is constant, as shown in Fig. 1(*e*), and remains so until plastic action begins. In this stage, the transfer of load at rivets again tends toward that called for by the elastic theory.

In Stage IV, yield occurs in the plates, rivets, or both; and slip again increases more rapidly than load. At the beginning, this increase in rate is caused in over-riveted joints by yielding of the plate material, and in under-riveted joints by yielding of the rivets; in either case there is no well-defined proportional limit. For other materials or structural members which may not

have a well-defined yield, often a "useful limit point" (U.L.P.) is taken as that point on the stress-strain curve at which the rate of deformation is twice that below the proportional limit. Similarly, for the under-riveted joints of this investigation a load called the "effective rivet yield" (E.R.Y.) was taken as that where the rate of slip with respect to load was twice that in Stage III. In a balanced joint, there would be uncertainty as to whether the useful limit point or the effective rivet yield is reached first.

In the latter part of the yield range, rivets in the end rows of a number of the joints tested in this investigation sheared before the maximum load had been reached. The rivets in these joints were $\frac{7}{8}$ in. in diameter. Measurements of slip between plates at sections through the rivets indicated that premature rivet failure occurred when the detrusion was about 0.3 in.

It seems logical to suppose that in Stage IV, while the steels are deforming plastically, the partition of load among rivets tends to become equalized. The premature rivet failures caused by excessive strain cannot necessarily be interpreted as indicative of higher-than-average stresses in rivets near the ends of a joint.

Description of Specimens

There were forty pairs of joints, which are herein considered in three principal groups, as shown in Table 1. The specimens are identified by a

TABLE 1.—Classification of Test Specimens

Series	Number of pairs	Figure number	Kind of splice	Number of plates	Designed to fail in:	Variables
C, D	16	3	Lap	1	Plate	Net width; end pitch; plate thickness.
A	15	6	Lap	1, 2, and 3	Rivets	Size of joint; rivet steel; plate steel.
B	3	7	Lap	1	Rivets	Pitch.
F	6	14	Butt, shingle	1, 2, and 3	Plates	Size of joint; butt *vs.* shingle splices.

series of letters and numbers. The first letter indicates the test series; the second is the initial of the plate steel (carbon, silicon, or nickel); and, the third is the initial of the rivet steel (carbon or manganese). For example, ASC denotes those specimens of Series A which have silicon-steel plates and carbon-steel rivets. The details of the specimens are shown in Figs. 2, 3, and 4.

In order not to unbalance the design of the joints more than necessary, when failure in the plates was desired the maximum expected strength of the plates was made equal to the minimum expected strength of the rivets. Conversely, when failure in the rivets was desired the maximum expected strength of the rivets was made equal to the minimum expected strength of the plates. The range in expected unit strengths, established from pilot tests at the University of Illinois,[8] is shown in Table 2. Table 2 also shows the working stress assigned to the various steels. Unless otherwise stated, the working load of a joint is taken as that which produces the working stress at the net section of the plates, even for joints designed to fail in the rivets. In the design of the

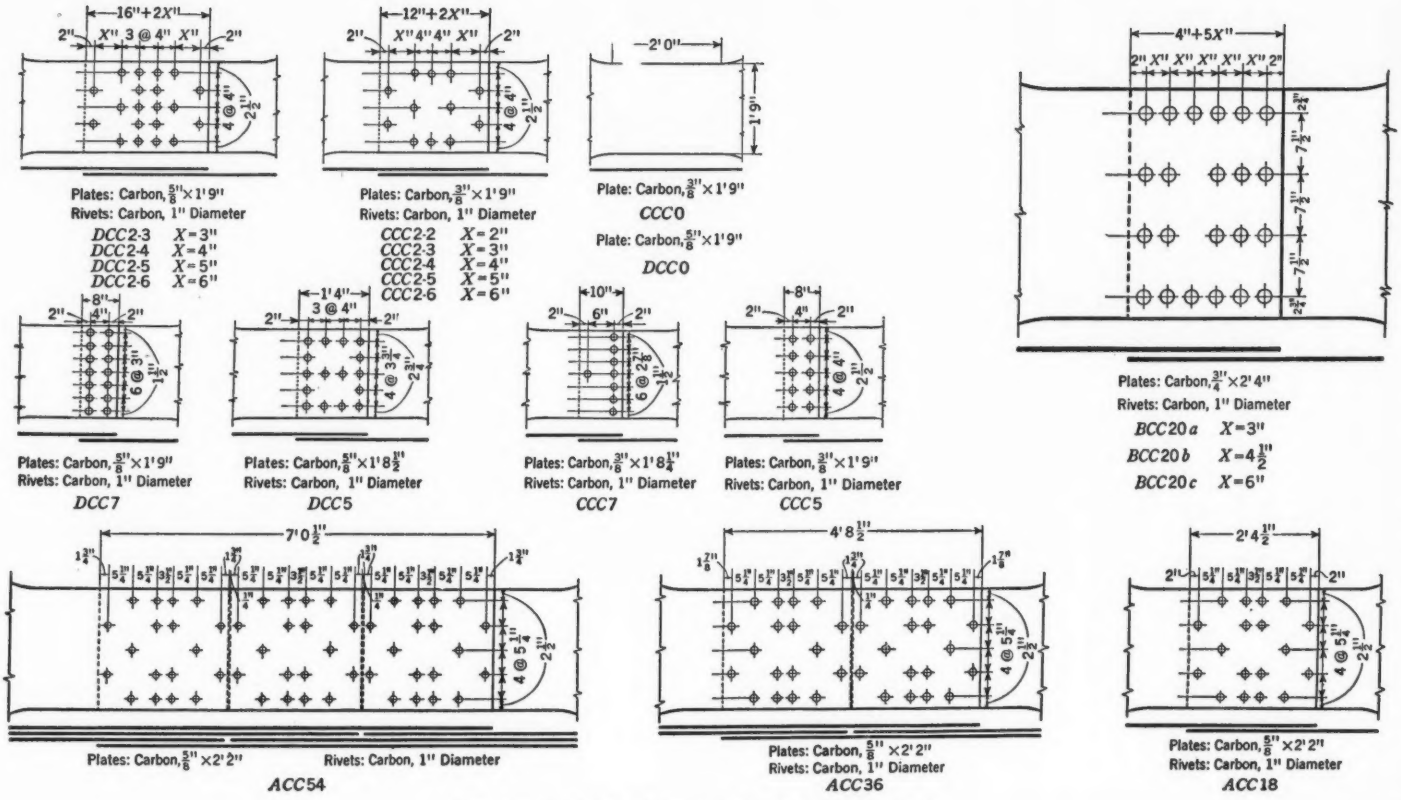
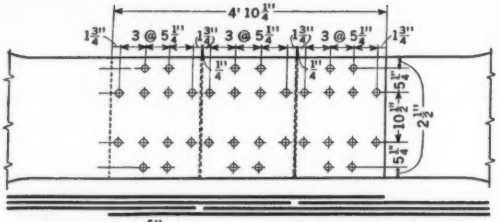
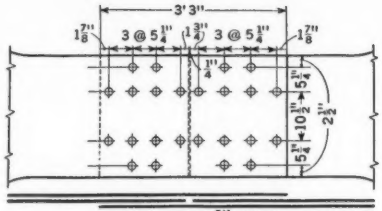
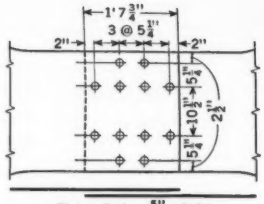
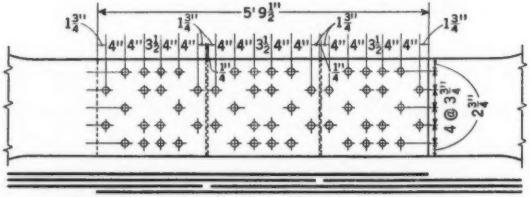


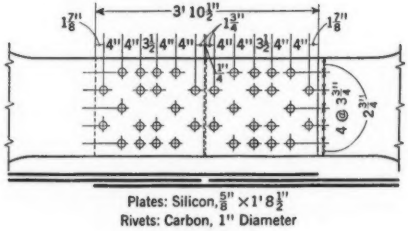
FIG. 2.—DETAILS OF SPECIMENS; SERIES A, B, C, AND D

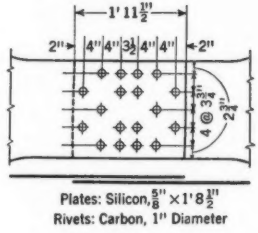


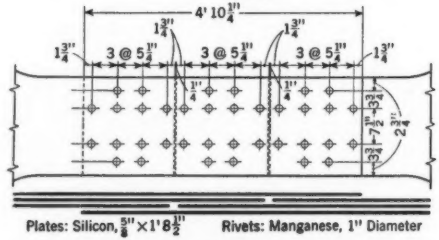


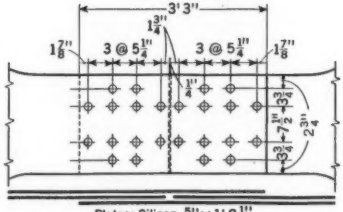












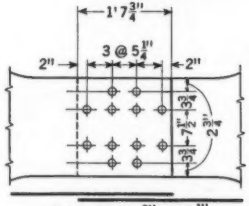


FIG. 3.—DETAILS OF SPECIMENS; SERIES A

FIG. 4.—DETAILS OF SPECIMENS: SERIES A AND F

specimens, the net section was taken through the end row of rivets. In computing the area of net section, the diameter of the rivet hole was taken as the nominal diameter of the rivet, plus $\frac{1}{8}$ in. In computations involving rivet shear areas, the nominal diameter of the rivet was employed.

TABLE 2.—RANGE IN EXPECTED UNIT STRENGTH, AND WORKING STRESS

Steel	UNIT STRESS, IN KIPS PER SQUARE INCH					
	(a) Tension in Plate			(b) Shear in Rivets		
	Range of Unit Strength:		Working stress	Range of Unit Strength:		Working stress
	From	To		From	To	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Carbon	60	72	22	53	61	15
Silicon	77	93	28	....	....	....
Nickel	90	110	34	....	....	....
Manganese	....	....	....	71	92	20

Materials.—The steels were rolled at the Homestead plant of the Carnegie-Illinois Steel Corporation to the specifications of the San Francisco-Oakland Bay Bridge, which were similar to the corresponding specifications of the American Society for Testing Materials. All of the materials for the specimens met the specification requirements. The chemical properties of the steels were as listed in Table 3. The yield strength and ultimate strength of the mill coupons taken from plates are given subsequently in the tables showing the results of tests on joints (Tables 5, 9, and 17).

TABLE 3.—CHEMICAL PROPERTIES OF STEEL
(Percentages of Total)

Chemical content	PLATE STEEL			RIVET STEEL	
	Nickel	Silicon	Carbon	Carbon	Manganese
Carbon	0.29 to 0.30	0.30 to 0.37	0.17 to 0.22	0.13	0.22
Manganese	0.63 to 0.65	0.73 to 0.77	0.53 to 0.55	0.45	1.35
Silicon	0.19 to 0.24	0.23 to 0.31	....	....	....
Nickel	2.91 to 3.03	....	....	....	....
Phosphorus	0.027 to 0.035	0.020 to 0.028	0.017 to 0.031	0.020	0.026
Sulfur	0.028 to 0.033	0.031 to 0.047	0.031 to 0.041	0.033	0.021
Copper	0.22	0.25 to 0.27	0.24 to 0.30	0.26	0.30

The physical properties of the rivet steel are shown in Table 4. The specimens were fabricated at the Ambridge plant of the American Bridge Company in accordance with the best commercial practice. For the carbon-steel and the silicon-steel plates, the rivet holes were sub-punched with a die of diameter $\frac{1}{8}$ in. less than the nominal diameter of the rivet, and the rivet holes were reamed to a diameter $\frac{1}{16}$ in. larger than that of the rivet. For the nickel-steel plates, the solid plates were assembled and the rivet holes were drilled to a diameter $\frac{1}{16}$ in. larger than that of the rivet. Rivets were driven with a hydraulic machine.

The temperature of carbon-steel rivets at the start of driving ranged from 1 400° F to 1 800° F with an average of 1 570° F. For manganese-steel rivets the range was 1 400° F to 1 900° F and the average 1 680° F.

TABLE 4.—PHYSICAL PROPERTIES OF RIVET STEELS

Specimen	Property	Carbon steel		Manganese steel	Annealed manganese steel
Diameter of specimen, in inches		⅞	1	1	1
Rivet rod stock*	Percentage: Elongation in 8 inches	31.0	33.0	24.1	27.6
	Reduction of area	58.0	58.8	55.5	62.7
	Strength, in Kips Per Square Inch: Yield strength	39.9	39.6	54.6	49.2
	Tensile strength	58.0	57.2	81.0	74.7
Non-driven rivets†	Shearing strength in double shear in kips per square inch	....	42.1	54.5	....

* Tested at the mill. † Tested at the University of Illinois.

METHODS OF TESTING; INTERPRETATION OF DATA

Before each specimen was tested, gage lines for strain-gage observations were established at selected locations on the faces and edges of the plates; dials to determine the slip between adjacent plates were placed at one or more points along each edge; reference lines were scribed across each edge at intervals throughout the length; and, a coat of white brittle paint was applied to the faces of the plates.

The specimens were tested in the Southwark-Emery hydraulic testing machine at the University of California. Before the lower end of the specimen was gripped in the testing machine, initial strain-gage observations were taken. Up to a stress greater than the working stress but less than the yield stress, the load was applied in four or five increments, at a slow rate (approximately 500 lb per sq in. of gross section per minute). During the application of these load increments, observations of slip were made by means of the dial gages. After each increment of load had been applied, the load was maintained constant while strain-gage observations were made.

Above the highest load at which strains were measured, the load was applied at a faster rate, usually about 2 500 lb per sq in. of gross section per minute, up to the maximum load. During this final period of loading, the extent of plate scaling was recorded at intervals by means of photographs; written record was made of the progress of paint chipping from the rivet heads and of local failure of individual rivets or plate material; and, for some specimens the slip was observed by means of the dial gages up to about two-thirds of the estimated maximum load.

Determination of Yield Strength.—As load was applied, any marked yield or sudden slip in a specimen or its connections was reflected by a halt or lag in movement of the load-indicating gages of the hydraulic testing machine. For forty-two of the specimens, a semi-autographic record of the relation between

load and time was obtained; for the other specimens, this relation was plotted from the log of the tests. On the load-time diagrams the successive loads at which various parts of the plates scaled were also noted. The yield load of the joint was taken as the first load at which the combined evidence of load lags and plate scaling was sufficiently marked so that it represented clearly the general behavior of the joint rather than minor sudden slips in the joint or in the pulling heads. Many of the joints did not exhibit a well-defined yield, and for seven of the joints no yield load could be assigned (Table 5, Specimens ASC54-2, ASM24-2, ASM36-2, ANM12-1, ANM24-2, ANM36-2, and BCC-20b-1). The yield loads determined as just described were probably close to those which might be determined from the load-elongation relations for the joint as a whole.

Attention is directed to the fact that herein the term "yield" does not indicate the same property as that reported for small joints by some investigators who considered yield as the load at which marked or sudden slip between plates first occurred.

Many of the joints with carbon-steel plates exhibited local scaling at loads appreciably below the yield load.

Usually, scaling of the plate began near rivets in the end row; and, for some specimens, the scaling through the yield range occurred largely outside the joint. It appears, therefore, that the yield strength, as determined, is a function of the stress conditions in the plate.

Determination of Effective Rivet Yield.—For joints failing in the rivets, it is believed that the load at which general rivet yielding occurs is of greater significance than the yield load based on plate action as just described. Herein a load called the "effective rivet yield" was determined, from the load-slip diagrams, as the point where the rate of slip with respect to load became twice the constant value it had attained while the joint was behaving elastically (Stage III, Fig. 1(*d*)). For the determination of this load, the slip measured at the middle of the joint was used; values obtained from slip measured at the ends of the joints were practically the same as those obtained from the middle slip. This load is analogous to the "useful limit point" used to define the practical proportional limit in materials or structural members which may not have a definite yield point. It was determined for all of the joints which had an effective rivet yield within the range of the slip measurements.

Use of Bending Frame.—The first three tests on lap joints disclosed that even at low loads the strains due to bending exceeded those due to direct stress, and that within the range of working loads the yield point of the steel was exceeded at critical locations. To reduce the bending within the length of the splice, the "bending frame" shown in Fig. 5 was used on most of the remaining lap joints (see Column (15), Table 9). After each increment of load had been applied to a specimen, the frame was adjusted until the bending within the length of the splice was reasonably small, as indicated by observations with a "curvemeter." The frame was removed after the last series of strain-gage observations had been completed.

Although the bending frame served its general purpose satisfactorily, the joints, as fabricated, were slightly warped and buckled both longitudinally and

transversely, with the result that considerable local differences in bending stress were induced by loading and by application of the frame.

Determination of Curvature.—The curvemeter used to determine curvature of the plate consisted of a rigid bar having at each end a leg which was applied to a gage point on the plate. At the center of the bar, a dial gage was mounted so that its plunger was in contact with the plate midway between the gage points. In order to convert the observed deflections into terms of bending strain at the surface of the plate, it was assumed that the elastic curve of the bent plate was a circular arc and that the neutral surface of the plate was midway between its faces.

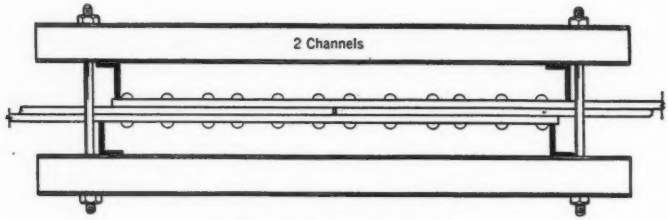


FIG. 5.—BENDING FRAME

Strain-Gage Measurements.—For most of the observations, 2.5-in., fulcrum-plate, strain gages were used; but for certain observations a gage length of 5, 10, or 20 in. was used. For a gage length of 2.5 in. the strain corresponding to the least reading by estimation was equivalent to a stress in the steel of 120 lb per sq in.

Many of the lap joints were curved while under load, even when the bending frame was used. The amount of bending within the length of the 2.5-in. strain-gage lines on the faces of the plates was determined, and curvature corrections were applied to the strain-gage observations.

With respect to the length of the joint, the longitudinal strain-gage lines were generally: (1) In the gross section of the plate outside of the splice; (2) in each space between rivet rows; and (3) spanning the rivet rows. Between rivet rows, for the single-plate and double-plate lap joints, in general, the gage lines were closely spaced on one-half of one face, and skeletonized on the remaining half of that face and on the opposite face. Corresponding gage lines were established on each edge of each plate. For the triple-plate lap joints and the butt joints, the observations were confined almost entirely to the edges of the plates. Transverse strains were measured only on the solid plates of Series C and D.

Slip Measurements.—The "slip," or relative movement of adjacent plates at a given section, was determined by means of dial gages, at the edges of the plates, by means of which movements of 0.0001 in. could be measured. Each gage on one plate was actuated by a lug attached to an adjacent plate. For very short, single-plate lap joints, the slip was measured at the middle of length; for the other single-plate lap joints, the slip at the ends was measured also. For multiple-plate joints, as a rule, the slip was measured at points where it was

considered that minimum and maximum movement would occur in each single lap which composed a section of the joint as a whole. The slip gages were fastened to the plates at the middle of their thickness.

In many cases, sudden slips occurred during the progress of load application. In the majority of instances, after sudden slip occurred, the rate lagged temporarily, so that after an additional few thousand pounds had been applied the load-slip relation became coincident with what it would have been had the sudden slip not occurred. In these cases, the over-all characteristics were not altered by these erratic movements, and continuous curves are presented herein to show the general load-slip relation.

Although the middle slip is perhaps a characteristic of a joint acting as a whole, the end slip is considered significant with regard to the work done on the rivets in the end row. It is obvious that the distance of the end slip gage from the middle of the joint affects the ratio of end slip to middle slip. In making comparisons between joints, any differences in this distance should be taken into account.

Determination of Unit Elongation.—The elongation of a riveted joint under load includes both the strain in the plates and the slip between the plates. For use in determining the elongation, generally, a continuous chain of 2.5-in. and 10-in. strain-gage lines extended along each edge of the specimen. Furthermore, for some of the longer joints the over-all change in length was measured with an attached extensometer. For computing the unit elongation, the length of the joint was taken as the distance between ends of the plates forming the splice. The ratio of unit elongation of the joint as a whole to the unit elongation of the gross section of the plate, at the working load of the joint, is herein called the "elongation ratio." Although the unit elongation includes plastic deformation and slip as well as elastic deformation, it may be used as a basis for computing an effective modulus of elasticity of the joint.

The modulus of elasticity of the plates used in the joints was based upon strains measured on the plates outside the splice. The modulus generally ranged from 28 to 31 million lb per sq in.; the average was 29.7 million lb per sq in.

Computation of Joint Efficiency.—The efficiency was taken as the ratio of the stress in the gross section at ultimate load to the tensile strength of corresponding mill coupons. In this investigation, the efficiency was computed only for the specimens that failed in the plates.

Computation of Rivet Strength Ratio.—In so far as the performance of the rivet materials is concerned, a factor herein called the "rivet strength ratio" is of more significance than joint efficiency based on the plates which, for joints failing in the rivets, merely reflects the conditions of a particular design. The rivet strength ratio evaluates the performance of the rivets in a joint with respect to their potential shearing strength, as follows:

$$\text{Rivet strength ratio} = \frac{\text{Average shearing resistance per rivet in a joint}}{\text{Shearing strength of non-driven rivets in double shear}} \quad \ldots\ldots (1)$$

The ratio was computed only for the specimens that failed in the rivets.

TABLE 5.—TESTS ON LAP JOINTS DESIGNED

Specimen No. (for details, see Fig. 2)¶	Plate (Carbon Steel)				Rivets (One Inch; Carbon Steel)			Mill Coupons (Kips per Square Inch)		Specimens	
	Thickness, in inches	Width, in inches	Area, in square inches		Total number	Number in first row	End pitch, in inches	Yield strength	Tensile strength	End slip at rivet working load, in inches	Ratio of end slip to middle slip*
			Gross	Net (first row)							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
CCC0-1	⅜	21	7.77	7.77	0	....	....	44.1	68.3	....	....
CCC0-2	⅜	21	7.77	7.77	0	....	....	....	....	....	....
Average			7.77	7.77				44.1	68.3	....	....
CCC2-2-1	⅜	21	7.80	6.96	12	2	2	42.9	69.3	0.0070	1.6†
CCC2-2-2	⅜	21	7.77	6.93	12	2	2	43.9	69.9	0.0080	1.2
Average			7.78	6.95				43.4	69.6	0.0075	....
CCC2-3-1	⅜	21	7.77	6.94	12	2	3	44.0	68.2	0.0107	1.2
CCC2-3-2	⅜	21	7.79	6.95	12	2	3	44.1	68.1	0.0105	1.3
Average			7.78	6.95				44.1	68.2	0.0106	1.3
CCC2-4-1	⅜	21	7.76	6.93	12	2	4	43.7	68.4	0.0105	1.5
CCC2-4-2	⅜	21	7.70	6.87	12	2	4	44.0	70.4	0.0091	1.4
Average			7.73	6.90				43.9	69.4	0.0098	1.5
CCC2-5-1	⅜	21	8.03	7.17	12	2	5	43.1	69.7	0.0122	1.5
CCC2-5-2	⅜	21	8.04	7.18	12	2	5	43.7	70.3	0.0047	2.0
Average			8.04	7.18				43.4	70.0	0.0085	1.8
CCC2-6-1	⅜	21	7.98	7.13	12	2	6	43.4	69.8	0.0128	1.7
CCC2-6-2	⅜	21	8.01	7.16	12	2	6	43.9	69.9	0.0086	1.5
Average			8.00	7.15				43.7	69.9	0.0107	1.6
CCC5-1	⅜	21	8.04	5.89	10	5	....	41.2	68.2	0.0080	....
CCC5-2	⅜	21	8.07	5.91	10	5	....	40.8	67.6	0.0098	....
Average			8.06	5.90				41.0	67.9	0.0089	....
CCC7-1	⅜	20½	7.80	4.77	8	7	....	42.3	67.4	0.0059	....
CCC7-2	⅜	20½	7.84	4.79	8	7	....	42.1	68.4	0.0097	....
Average			7.82	4.78				42.2	67.9	0.0078	....
DCC0-1	⅜	21	13.12	13.12	0	....	....	38.1	68.1	....	....
DCC0-2	⅜	21	13.16	13.16	0	....	....	....	....	....	....
Average			13.14	13.14				38.1	68.1	....	....
DCC2-3-1	⅜	21	13.08	11.68	20	2	3	39.3	68.1	0.0072	2.0
DCC2-3-2	⅜	21	13.11	11.71	20	2	3	38.5	67.3	0.0034	2.1
Average			13.10	11.70				38.9	67.7	0.0053	2.1
DCC2-4-1	⅜	21	13.18	11.77	20	2	4	38.2	68.4	0.0078	2.7
DCC2-4-2	⅜	21	13.17	11.76	20	2	4	38.0	67.6	0.0077	3.0
DCC2-4-3	⅜	21	13.15	11.74	20	2	4	38.6	67.2	0.0080	2.5
DCC2-4-4	⅜	21	13.17	11.76	20	2	4	39.6	69.2	0.0070	1.8
Average			13.17	11.76				38.6	68.1	0.0076	2.5
DCC2-5-1	⅜	21	13.02	11.63	20	2	5	40.5	67.6	0.0082	2.3
DCC2-5-2	⅜	21	13.05	11.65	20	2	5	39.2	67.8	0.0058	2.2
Average			13.04	11.64				39.9	67.7	0.0070	2.3
DCC2-6-1	⅜	21	12.97	11.58	20	2	6	38.5	67.0	0.0070	2.6
DCC2-6-2	⅜	21	12.99	11.60	20	2	6	38.4	67.3	0.0075	2.1
Average			12.98	11.59				38.5	67.2	0.0073	2.4
DCC5-1	⅜	20½	12.79	9.28	16	5	....	38.5	67.9	0.0044	....
DCC5-2	⅜	20½	12.80	9.29	16	5	....	37.5	67.3	0.0050	....
Average			12.80	9.29				38.0	67.6	0.0047	....
DCC7-1	⅜	21	13.09	8.18	14	7	....	38.3	67.7	0.0029	....
DCC7-2	⅜	21	13.09	8.18	14	7	....	37.5	67.8	0.0033	....
Average			13.09	8.18				37.9	67.8	0.0031	....

* At highest maintained load (25 kips per sq in. in plate, except as noted by reference (†)); at this load the ratios were practically constant. † At 20 kips per sq in. in plate. ¶ For all specimens CCC2 and DCC2, bending frame (Fig. 5) used up to stress of 25 kips per sq in. at net section of plate.

TO FAIL IN PLATES; SERIES C AND D

SPECIMENS (*Continued*)

Elongation ratio at plate working load‡	At Yield Load			At Ultimate Load					
	Stress in plate, in kips per square inch		Efficiency, percentages‖	Load, in kips	Stress in plate, in kips per square inch		Efficiency, percentages‖	Effective reduction in width due to rivets, in inches	
	Gross	Net			Gross	Net		By test (see Equation (2))	By formula (see Equation (3))
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
....	37.4	37.4		491	63.2	63.2			
....	38.3	38.3		480	61.8	61.8			
....	37.9	37.9	86.0	485	62.5	62.5	91.5	....	....
1.33	38.2	42.8		399	51.2	57.3			
1.50	37.7	42.3		390	50.2	56.3			
1.42	38.0	42.6	87.5	394	50.7	56.8	72.8	5.71	4.87
1.63	36.7	41.1		388	49.9	55.9			
1.54	37.0	41.4		391	50.2	56.3			
1.59	36.9	41.3	83.7	389	50.1	56.1	73.4	5.59	3.94
1.38	38.7	43.3		400	51.5	57.7			
1.38	38.2	42.8		403	52.3	58.7			
1.38	38.5	43.0	87.7	401	52.0	58.2	75.0	5.25	2.62
1.41	37.5	42.0		428	53.3	59.7			
0.99	37.4	41.9		410	51.0	57.1			
1.20	37.5	42.0	86.4	419	52.2	58.4	74.4	5.38	2.25
1.31	36.8	41.2		420	52.6	58.9			
1.31	37.2	41.6		417	52.1	58.2			
1.31	37.0	41.4	84.7	418	52.4	58.6	74.9	5.27	2.25
2.72	35.5	48.4		423	52.6	71.8			
3.11	35.7	48.7		413	51.2	69.9			
2.92	35.6	48.6	86.8	418	51.9	70.9	76.5	4.94	5.63
2.46	35.9	58.7		356	45.6§	74.6§			
3.32	35.7	58.5		360	45.9§	75.2§			
2.89	35.8	58.6	84.8	358	45.8§	74.9§	67.5§	<6.60§	7.88
....	34.3	34.3		839	63.9	63.9			
....	35.3	35.3		841	63.9	63.9			
....	34.8	34.8	91.4	840	63.9	63.9	93.8	....	....
0.99	33.0	37.0		691	52.8	59.2			
0.87	33.6	37.6		703	53.6	60.0			
0.93	33.3	37.3	85.6	697	53.2	59.6	78.6	4.50	3.94
1.03	33.3	37.2		701	53.2	59.6			
0.93	33.0	36.9		698	53.0	59.4			
1.01	33.5	37.5		710	54.0	60.5			
1.05	33.7	37.8		717	54.4	61.0			
1.01	33.4	37.4	86.6	707	53.7	60.1	78.8	4.45	2.62
0.98	32.4	36.3		686	52.7	59.0			
1.00	33.3	37.3		695	53.3	59.7			
0.99	32.9	36.8	82.5	691	52.9	59.3	78.1	4.60	2.25
0.97	32.2	36.1		669	51.6	57.8			
0.93	32.8	36.7		684	52.7	59.0			
0.95	32.5	36.4	84.4	677	52.2	58.4	77.7	4.68	2.25
1.33	32.1	44.3		654	51.1	70.5			
1.36	32.1	44.2		671	52.4	72.2			
1.35	32.1	44.3	84.5	663	51.8	71.4	76.6	4.80	5.63
1.98	31.6	50.6		624	47.7§	76.3§			
2.18	31.7	50.7		622	47.5§	76.0§			
2.08	31.7	50.7	83.6	623	47.6§	76.2§	70.2§	<6.26§	7.88

‡ Ratio of unit elongation of joint to unit elongation of plate (gross section). § Failure in rivets.
‖ Ratio of stress in gross section of plate to corresponding strength of mill coupons.

Tests of Lap Joints Designed to Fail in Plates; Series C and D

The principal purpose of these tests on lap joints was to provide information regarding the efficiency of riveted tension members as influenced by: (1) The end pitch, or distance between the first and second transverse rows of rivets; (2) the effective net width of plate as controlled by the number of holes at a given section; and (3) the bearing stresses as controlled by thickness of plate.

The details of the specimens are shown in Fig. 2. All plates and rivets were of carbon steel. The rivets were 1 in. in diameter. The plates were nominally 21 in. wide, and were $\frac{3}{8}$ in. thick for Series C and $\frac{5}{8}$ in. thick for Series D. The $\frac{3}{8}$-in. material was thinner than required by standard specifications to develop the single-shear value of the rivets. Solid plates were included to provide data on the strength and efficiency (based on the strength of coupons) of wide plates without holes. The results of the tests are summarized in Table 5.

Tests of Solid Plates.—The efficiency at ultimate load, based on the strength of coupons tested at the mill, was 92% for the $\frac{3}{8}$-in. solid plates and 94% for the $\frac{5}{8}$-in. plates (Table 5). At the yield load, the efficiencies were 86% and 91%, respectively. These results are in agreement with those of other investigators.[7]

For loads up to 30 kips per sq in., longitudinal strains were the same at the middle of the plate as at the edge. For one $\frac{3}{8}$-in. specimen, the strains were measured at a stress of 35 kips per sq in., at which load the longitudinal strain was 11% greater at the middle of the plate than at the edge.

The specimens exhibited a marked yield at an average stress of 37.9 kips per sq in. for the $\frac{3}{8}$-in. plates and 34.8 kips per sq in. for the $\frac{5}{8}$-in. plates. The brittle paint on the specimens began to flake at stresses at, or slightly below, the yield point. This flaking exhibited the characteristic Lueders lines which started at the middle of the plate and progressed toward the edges as the loading was increased.

The plates failed by tearing apart at mid-length. For three specimens, a short transverse crack started at the middle of width of the plate; for the fourth specimen, the first crack started at one edge. The tendency for failure to start in the middle is attributed to the unequal distribution of longitudinal strain. The average elongation of the plates at failure was 29% in a 20-in. length. The reduction of area at failure was 28% for one $\frac{3}{8}$-in. plate and 35% for one $\frac{5}{8}$-in. plate.

Lateral and longitudinal strain-gage measurements were made on a grid of sixteen 5-in. squares which had been established on one $\frac{3}{8}$-in., and one $\frac{5}{8}$-in. solid plate. Within the working range, Poisson's ratio was found to be 0.28 for plates of both thicknesses. After failure, the ratio of lateral to longitudinal set was found to be 0.38 for the $\frac{3}{8}$-in. plate and 0.40 for the $\frac{5}{8}$-in. plate. These values are of interest in connection with the large transverse component of shear induced in widely spaced rivets in the end rows of long joints.

Effect of Number of Holes at a Given Section.—The effect of the number of rivets in the end row is shown in detail in Table 5 and summarized in Table 6. Of the riveted specimens, the highest total strength and efficiency was attained by the joints having five rivets in the end row at a gage distance of about 4 in. In the specimens having seven rivets in the end row at a gage distance of

about 3 in., failure took place in the rivets rather than in the plates, although it is believed that plate failure was imminent at the time the joints ruptured.

For either of the sets of riveted joints, the calculated average stress in the net section at ultimate load was greater than that for the solid plates, as shown in Table 6. This difference is attributed in part to the difference between the

TABLE 6.—EFFECT OF NUMBER OF RIVETS IN END ROW; SERIES C AND D

Plate thickness, in inches	Number of Rivets		Symbol	Gage, in inches	Location of failure	Ratio of net to gross area	Stress in Net Section at Ultimate Load		Efficiency percentages
	In first row	Total					Kips per square inch	Percentage of strength of solid plates	
3/8	0	0	CCC 0	....	Plate	1.00	62.5	100	91.5
	5	10	CCC 5	4	Plate	0.73	70.9	113	76.5
	7	8	CCC 7	2½	Rivets	0.61	74.9*	120*	67.5*
5/8	0	0	DCC 0	....	Plate	1.00	63.9	100	93.8
	5	16	DCC 5	3¼	Plate	0.73	71.4	112	76.6
	7	14	DCC 7	3	Rivets	0.63	76.2*	119*	70.2*

* Strength of plates was higher than value shown, since failure was in rivets.

assumed ($1\frac{1}{8}$ in.) and the actual ($1\frac{1}{16}$ in.) diameter of the rivet holes, and in part to what has been termed "the grooving effect" of notched plates.[10, 11] The joints having the lower bearing stresses ($\frac{5}{8}$-in. plates) exhibited the higher efficiencies.

Effect of End Pitch.—The detailed test results for the group of joints to determine the effect of end pitch on strength and efficiency are shown in Table 5. All specimens failed in the plates along lines that passed through the rivet holes of the end row, as illustrated in Fig. 6. For the shorter end pitches, the fractures generally followed a zigzag course, passing through rivet holes in the first and second rows. The tendency for the fractures to extend through both end rows of rivets was less for the $\frac{3}{8}$-in. plates than for the $\frac{5}{8}$-in. plates, possibly because of higher bearing stresses in the $\frac{3}{8}$-in. plates. Furthermore, the $\frac{3}{8}$-in. plates exhibited a pronounced bulge at the end, opposite each end rivet, and the extent of this bulging was greater than that in any of the other specimens tested.

Fig. 7 shows a typical joint, with $\frac{5}{8}$-in. plates, after failure. The reduction in percentage of paint scaling from top to bottom of the splice reflects the transfer of stress from the near, to the far, plate. The characteristic Lueders lines where the mill scale and white paint have flaked off show the direction of maximum shear in the plate between rivets. The scaled areas at the front (upper) edges of the rivet heads indicate that the rivets have tilted forward as load was applied. At the bottom of the splice, it is seen that the slip between adjacent plates was about $\frac{3}{8}$ in. It is of interest to note also the extent to which the highly stressed section of the lower plate contracted laterally.

[10] "Johnson's Materials of Construction," by M. O. Withey and James Aston, John Wiley & Sons, Inc., New York, N. Y., Edition 7 (1930), pp. 663–666.

[11] "Strength of Materials," Part II, by S. Timoshenko, D. Van Nostrand Company, New York, N. Y., 1930, pp. 673–674.

The efficiencies of the joints at ultimate load, shown in Table 5, Column (20), are summarized in Table 7. The maximum difference in strength of joints with a given plate thickness was only about 5 per cent. Within this range, the $\frac{3}{8}$-in. specimens having staggered end rivets with an end pitch of 2 to 6 in. were less efficient than those with zero end pitch (five rivets in the end

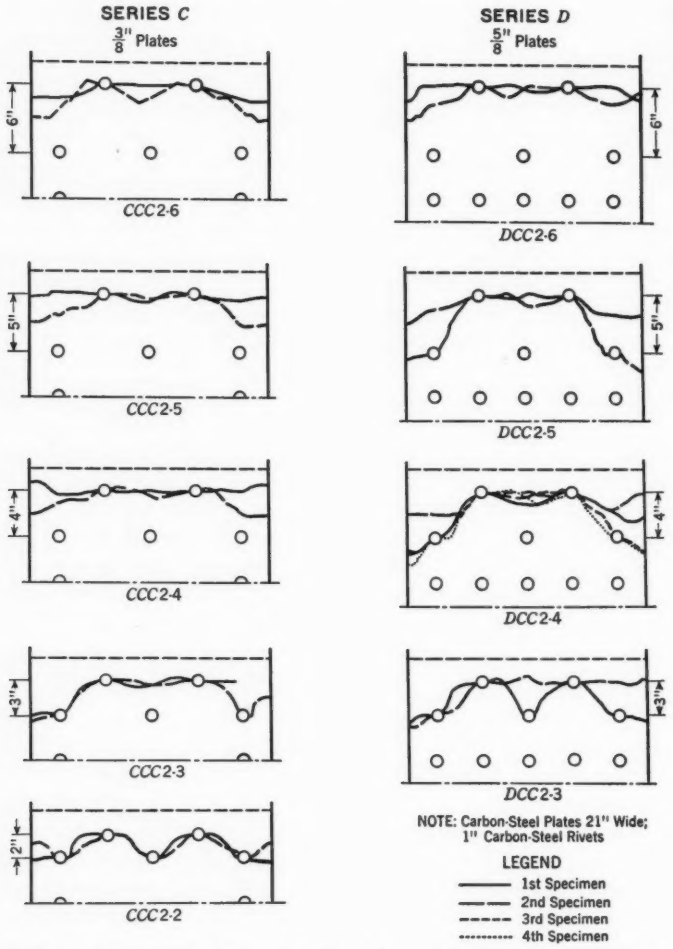


FIG. 6.—LOCATION OF FRACTURE; SERIES C AND D; VARIABLE END PITCH

row). The $\frac{5}{8}$-in. specimens with staggered end rivets were somewhat more efficient than those with zero end pitch. If plates thicker than $\frac{5}{8}$ in. had been used, it is possible that the relative strength of the joints with staggered rivets would have been somewhat higher.

For both thicknesses of plate there was a decrease in the efficiency of the joints when the end pitch exceeded 4 in. A possible explanation is that the

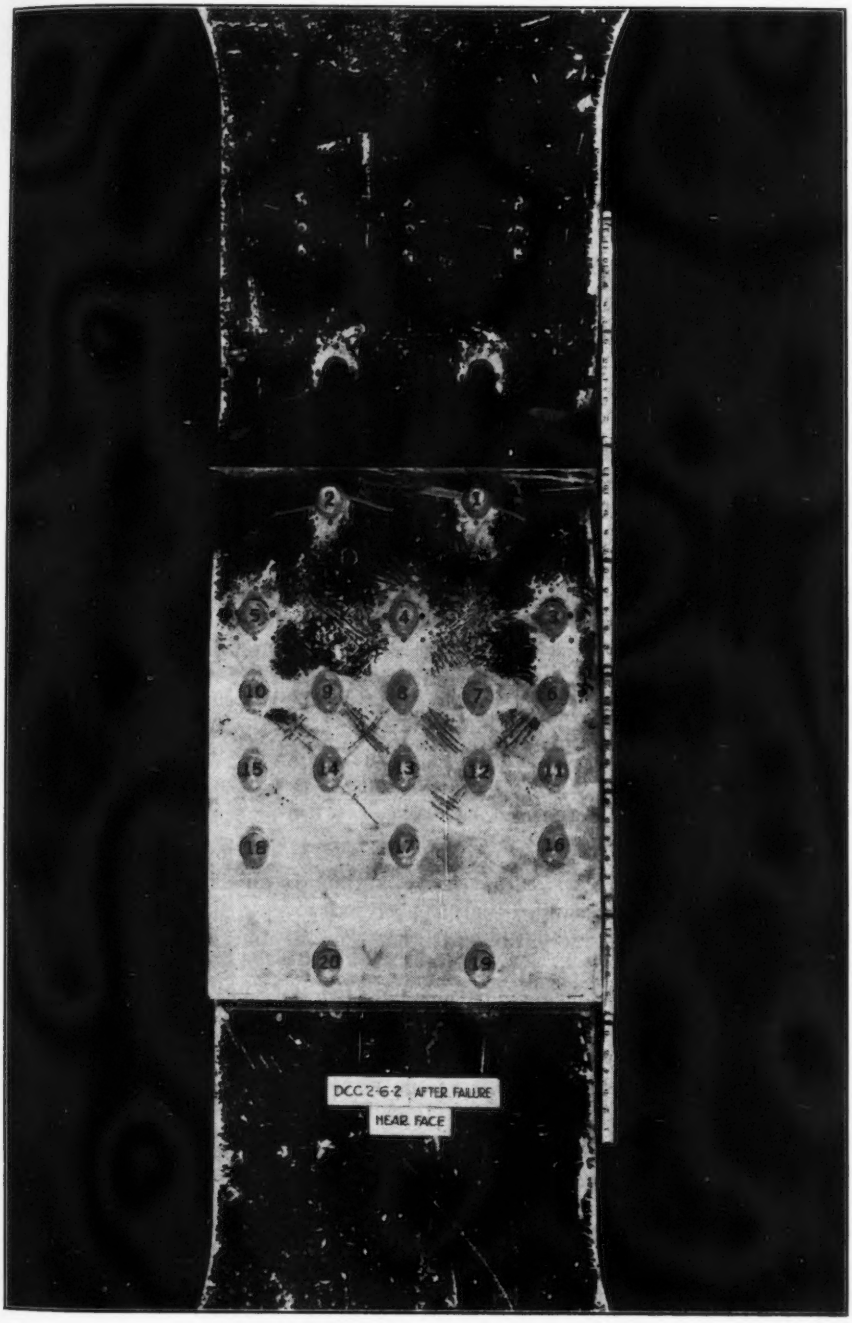


FIG. 7.—TYPICAL PLATE FAILURE; SERIES D

high concentrations of stress at the extreme rivets of the longer joints precipitated plate failure. The efficiency of specimens in Series C ($\frac{3}{8}$-in. plates) was less than that for specimens in Series D ($\frac{5}{8}$-in. plates), possibly because of the higher bearing stresses in the $\frac{3}{8}$-in. specimens.

TABLE 7.—EFFECT OF END PITCH; SERIES C AND D

Series	Plate thickness, in inches	Number of Rivets		Percentage Efficiency of Joint, for the Following End Pitch					
		End pitch, 6 inches to 2 inches	End pitch, 0 inches*	End pitch, 6 inches	End pitch, 5 inches	End pitch, 4 inches	End pitch, 3 inches	End pitch, 2 inches	End pitch, 0 inches
CCC	3/8	12	10	74.9	74.4	75.0	73.4	72.8	76.5
DCC	5/8	20	16	77.7	78.1	78.8	78.6	....	76.6

* Five rivets in end row; other joints have two rivets in end row and three rivets in second row.

Effective Net Section.—The results of a study to determine the effective reduction in width of plates due to rivet holes are shown in Table 5, Column (21). The calculation is based on the assumption that the potential strength of a joint is the product of the strength of the mill coupons and the gross cross-sectional area of the plate; this assumption is not strictly true, as shown by the fact that the efficiency of the solid plates CCC0 and DCC0 was only 92% and 94% respectively. The effective width of a riveted plate is the gross width times the efficiency. The effective reduction in width is then the gross width minus the effective width; that is (see Column (21), Table 5)

$$\text{Effective reduction in width} = \text{Width of plate} - \left(\text{Width of plate} \times \frac{\text{Efficiency at ultimate load}}{100}\right) \quad \ldots\ldots\ldots (2)$$

Corresponding values of actual reduction in width (assuming $1\frac{1}{8}$-in. holes) are $2\frac{1}{4}$ in. for two rivets, $4\frac{1}{2}$ in. for four rivets, and $5\frac{5}{8}$ in. for five rivets. It is seen that for all of the joints with widely spaced end rivets (Specimens CCC2 and DCC2) the effective reduction in width corresponds more nearly to five rivets than to two rivets.

The effective reductions in width calculated on the basis of a typical formula used in current specifications[12] are shown in Column (22), Table 5. The formula is:

$$x = 1 - \frac{s^2}{4\,g\,h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which x = the fraction of the hole to be deducted from b; s = the pitch, or the stagger; g = the gage of any two successive holes in a chain; and h = rivet diameter plus $\frac{1}{8}$ in. It is apparent that these requirements bear no relation to the results of the tests.

These results show that the use of fewer rivets in the end rows than in the interior rows does not make for the efficiencies that are calculated in accordance

[12] "Standard Specifications for Highway Bridges," Am. Assoc. of State Highway Officials, 1935.

with common practice. The yield strengths (Table 5, Column (15)) show no large or consistent effect of end pitch or number of rivet holes at a given section on yield of joint.

Tests of Lap Joints Failing in Rivets; Series A and B

As shown in Figs. 2, 3, and 4, the specimens of Series A consisted of fifteen pairs, which included three different lengths of lap joint for five different combinations of rivet and plate material, as listed in Table 8. The joints were

TABLE 8.—Characteristics of Specimens; Series A

Length of joint, in inches	Plates (all $\frac{1}{2}$ Inch)			Number of Rivets (all 1 Inch)	
	No.	Width in inches	Steel	Carbon steel	Manganese steel
20 to 29	1	26 20.5 18	Carbon Silicon Nickel	18 18	12 12 12
39 to 57	2	26 20.5 18	Carbon Silicon Nickel	36 36	24 24 24
58 to 85	3	26 20.5 18	Carbon Silicon Nickel	54 54	36 36 36

designed to fail in the rivets, and for a given length were designed to be of approximately equal strength. Lap, rather than butt, joints were used since within the capacity of the testing machine a greater variation in length of joint could be obtained by designing the specimens for single rather than double shear in the rivets. Lap joints also permitted the location of strain gages across the face of the main plates. The arrangement of the lap joints resulted in an eccentricity of $\frac{5}{8}$ in. for all joints. The uncertainties arising from, and the measures taken to correct for, the moments produced by this eccentricity have been mentioned previously herein (see headings, "Methods of Testing; Interpretation of Data: Use of Bending Frame," "Determination of Curvature" and "Strain-Gage Measurements").

The specimens of Series A were designed to furnish the following information:

(1) Effect of length of joint upon average shearing resistance of rivets;
(2) Effect of plate material upon average shearing resistance of rivets;
(3) Comparative behavior of carbon-steel and manganese-steel rivets;
(4) Partition of rivet shear among the several transverse rows of rivets;
(5) Relation between the average shearing resistance of rivets and the shearing strength of non-driven rivets; and,
(6) Desirable ratio of tensile stress in plates to shearing stress in rivets, for use in design of large riveted connections.

The specimens of Series B consisted of three pairs of lap joints with $\frac{3}{4}$-in. by 28-in. carbon-steel plates and twenty-two 1-in. carbon-steel rivets arranged in full rows of four rivets (Fig. 2). The rivet pitch throughout the joint was

TABLE 9.—Tests on Lap Joints Failing

Specimen No. (for details, see Figs. 2, 3, and 4)	Plates						Rivets (Diameter, One Inch)				Mill Coupons (Kips per Square Inch)		Specimens		
	Steel*	Width, in inches	Thickness, in inches	Number of plates	Area, in square inches		Steel*	Total number	Number in first row	Pitch, in inches	Yield strength	Tensile strength	Length of joint, in inches†	Bending frame (Fig. 5) used in working range	End slip at rivet working load, in inches
					Gross	Net (first row)									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
ACC18-1					16.08	14.69					41.3	69.7		No	0.0052
ACC18-2	C	26		1	16.13	14.73	C	18	2	5¼	39.5	68.2	28½	Yes	0.0037
Average					16.11	14.71					40.4	69.0			0.0045
ACC36-1					32.32	29.52					38.8	68.8		Yes	0.0038
ACC36-2	C	26		2	32.25	29.46	C	36	2	5¼	38.7	68.9	56½	Yes	0.0057
Average					32.29	29.49					38.8	68.9			0.0048
ACC54-1					48.97	44.73					39.4	68.5		No	0.0031
ACC54-2	C	26		3	48.99	44.75	C	54	2	5¼	39.4	67.9	84½	No	0.0048
Average					48.98	44.74					39.4	68.2			0.0040
ASC18-1					12.92	11.50					55.1	92.0		Yes	0.0075
ASC18-2	S	20½		1	12.93	11.51	C	18	2	4	54.5	90.8	23½	Yes	0.0075
Average					12.93	11.51					54.8	91.4			0.0075
ASC36-1					25.34	22.56					53.4	91.7		Yes	0.0075
ASC36-2	S	20½		2	25.70	22.88	C	36	2	4	51.3	90.8	46½	Yes	0.0094
Average					25.52	22.72					52.4	91.3			0.0085
ASC54-1					37.29	33.20					53.3	91.1		No	0.0092
ASC54-2	S	20½		3	38.69	34.45	C	54	2	4	51.7	91.2	69½	No	0.0084
Average					37.99	33.83					52.5	91.2			0.0088
ACM12-1					16.18	14.78					40.5	68.6		No	0.0125
ACM12-2	C	26		1	16.20	14.80	M	12	2	5¼	38.6	68.7	19¾	Yes	0.0099
Average					16.19	14.79					39.6	68.7			0.0112
ACM24-1					32.44	29.64					38.6	69.4		Yes	0.0079
ACM24-2	C	26		2	32.44	29.63	M	24	2	5¼	38.8	69.3	39	Yes	0.0102
Average					32.44	29.64					38.7	69.4			0.0091
ACM36-1					48.67	44.46					38.6	68.6		No	0.0075
ACM36-2	C	26		3	48.98	44.74	M	36	2	5¼	39.0	67.0	58¼	No	0.0093
Average					48.83	44.60					38.8	67.8			0.0084
ASM12-1					12.69	11.30					55.0	91.1		No	0.0125
ASM12-2	S	20½		1	12.69	11.29	M	12	2	5¼	54.8	92.0	19¾	Yes	0.0123
Average					12.69	11.30					54.9	91.6			0.0124
ASM24-1					25.11	22.35					51.1	92.0		Yes	0.0124
ASM24-2	S	20½		2	25.26	22.49	M	24	2	5¼	53.3	92.0	39	Yes	0.0117
Average					25.19	22.42					52.3	92.0			0.0121
ASM36-1					38.36	34.15					51.6	89.9		No	0.0109
ASM36-2	S	20½		3	38.02	33.84	M	36	2	5¼	52.6	90.8	58¼	No	0.0132
Average					38.19	34.00					52.1	90.4			0.0121
ANM12-1					11.20	9.80					65.2	107.2		Yes	0.0112
ANM12-2	N	18		1	11.19	9.79	M	12	2	5¼	66.1	106.2	19¾	Yes	0.0115
Average					11.20	9.80					65.7	106.7			0.0114
ANM24-1					22.20	19.43					67.8	104.4		Yes	0.0092
ANM24-2	N	18		2	22.23	19.45	M	24	2	5¼	67.7	105.9	39	Yes	0.0123
Average					22.22	19.44					67.8	105.2			0.0108
ANM36-1					33.45	29.27					66.1	105.6		No	0.0101
ANM36-2	N	18		3	33.45	29.27	M	36	2	5¼	63.3	105.8	58¼	No	0.0143
Average					33.45	29.27					64.7	105.7			0.0122
BCC-20a-1					20.66	17.34					35.6	68.8		Yes	0.0009
BCC-20a-2	C	28		1	20.68	17.36	C	22	4	3	35.2	68.5	19	Yes	0.0018
Average					20.67	17.35					35.4	68.7			0.0014
BCC-20b-1					20.52	17.23					37.0	66.0		Yes	0.0021
BCC-20b-2	C	28		1	20.70	17.37	C	22	4	4½	36.3	68.4	26½	Yes	0.0018
Average					20.61	17.30					36.7	67.2			0.0020
BCC-20c-1					20.49	17.19					37.5	65.4		Yes	0.0028
BCC-20c-2	C	28		1	20.49	17.20	C	22	4	6	36.9	65.8	34	Yes	0.0014
Average					20.49	17.20					37.2	65.6			0.0021
CCC7-1					7.80	4.77					42.3	67.4		No	0.0059
CCC7-2	C	20¼		1	7.84	4.79	C	8	7	6	42.1	68.4	10	No	0.0097
Average					7.82	4.78					42.2	67.9			0.0078
DCC7-1					13.09	8.18					38.3	67.7		No	0.0029
DCC7-2	C	21		1	13.09	8.18	C	14	7	4	37.5	67.8	8	No	0.0033
Average					13.09	8.18					37.9	67.8			0.0031

* C = carbon; S = silicon; M = manganese; and N = nickel.
† Measured from end to end of overlapping plates.
‡ In general, for the range in which this ratio was constant.
§ Ratio of unit elongation of joint to unit elongation of plates (gross section).

SPECIMENS (*Continued*)

Ratio of end slip to middle slip‡	Elongation ratio, at plate working load§	At effective rivet yield; shear in rivets		At Yield Load				At Ultimate Load				Number of rivets that failed before complete rupture
				Stress in plate, in kips per square inch		Shear in rivets			Shear in rivets			
		In kips per rivet	In kips per square inch	Gross	Net	In kips per rivet	In kips per square inch	Load, in kips	In kips per rivet	In kips per square inch	Ratio¶	
(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
2.06	1.04			33.9	37.1	30.3	38.6	783	43.5	55.4		**
2.09	1.08			34.3	37.5	30.7	39.1	790	43.9	55.9		**
2.08	1.09	25.0	31.8	34.1	37.3	30.5	38.9	787	43.7	55.7	1.33	
....	1.13			31.4	34.4	28.2	35.9	1431	39.8	50.6		2+**
1.80	1.15			32.3	35.3	28.9	36.8	1460	40.6	51.6		5
1.80	1.14	25.4	32.3	31.9	34.9	28.6	36.4	1446	40.2	51.1	1.22	
1.40	1.03			33.3	36.4	30.2	33.4	2082	38.6	49.1		7
1.72	1.04			32.0	35.0	29.0	37.0	2157	39.9	50.9		13
1.56	1.04	27.8	35.4	32.7	35.7	29.6	37.7	2120	39.3	50.0	1.19	
1.44	1.42			44.0	49.4	31.5	40.2	780	43.3	55.2		0
1.48	1.28			43.6	48.9	31.3	39.8	769	42.7	54.4		0
1.46	1.35	....	....	43.8	49.2	31.4	40.0	775	43.0	54.8	1.30	
....	1.29			46.4	52.1	32.6	41.6	1529	42.5	54.1		**
1.82	1.18			46.0	51.7	32.8	41.8	1520	42.2	53.8		3
1.82	1.24	25.3	32.3	46.2	51.9	32.7	41.7	1525	42.4	54.0	1.29	
1.45	1.16			44.3	49.7	30.6	38.9	2168	40.1	51.1		1+**
1.35	1.19			||	||	||	||	2163	40.1	51.0		6
1.40	1.18	25.3	32.2	44.3	49.7	30.6	38.9	2166	40.1	51.1	1.21	
1.56	1.96			34.4	37.6	46.3	59.0	730	60.8	77.5		0
1.20	2.04			34.3	37.6	46.3	59.0	699	58.3	74.2		0
1.38	2.00	35.7	45.4	34.4	37.6	46.3	59.0	715	59.6	75.9	1.39	
....	1.51			32.0	35.0	43.3	55.0	1437	59.9	76.2		**
1.45	1.68			34.0	37.2	45.9	58.5	1483	61.8	78.6		2+**
1.45	1.60	38.8	49.3	33.0	36.1	44.6	56.8	1460	60.9	77.4	1.42	
1.11	1.33			33.7	36.9	45.6	58.1	1948	54.1	68.9		12
1.37	1.36			31.6	34.6	43.0	54.8	2085	57.9	73.8		2
1.24	1.35	37.5	47.8	32.7	35.8	44.3	56.5	2017	56.0	71.4	1.31	
1.51	1.63			44.9	50.4	47.4	60.4	680	56.7	72.2		0
1.24	1.73			44.0	49.4	46.5	59.2	683	56.9	72.5		0
1.38	1.68	35.6	45.3	44.5	49.9	47.0	59.8	682	56.8	72.4	1.33	
....	1.51			47.5	53.4	49.7	63.3	1417	59.0	75.2		0
1.55	1.37			||	||	||	||	1522	63.4	80.7		0
1.55	1.44	37.3	47.5	47.5	53.4	49.7	63.3	1470	61.2	78.0	1.43	
1.20	1.30			42.2	47.4	45.0	57.3	2075	57.6	73.4		2
1.45	1.36			||	||	||	||	2032	56.4	71.9		2
1.33	1.33	35.8	45.6	42.2	47.4	45.0	57.3	2054	57.0	72.7	1.34	
1.40	1.56			||	||	||	||	713	59.4	75.6		0
1.22	1.56			54.1	61.8	50.4	64.2	695	57.9	73.7		0
1.31	1.56	32.9	41.9	54.1	61.8	50.4	64.2	704	58.7	74.7	1.37	
....	1.39			57.7	65.9	53.4	67.9	1498	62.4	79.4		0
1.67	1.44			||	||	||	||	1430	59.6	75.9		0
1.67	1.42	36.7	46.7	57.7	65.9	53.4	67.9	1464	61.0	77.7	1.43	
1.14	1.23			55.6	63.5	51.7	65.8	2125	59.0	75.2		0
1.60	1.40			||	||	||	||	2155	59.9	76.2		0
1.37	1.32	38.8	49.3	55.6	63.5	51.7	65.8	2140	59.5	75.7	1.39	
1.44	1.15			32.8	39.0	30.8	39.2	975	44.3	56.4		0
1.35	1.27			33.4	39.8	31.4	40.0	975	44.3	56.4		0
1.40	1.21	....	....	33.1	39.4	31.1	39.6	975	44.3	56.4	1.34	
1.12	1.30			||	||	||	||	1003	45.6	58.0		1
1.19	1.23			33.2	39.6	31.3	39.8	1023	46.5	59.2		0
1.16	1.27	....	....	33.2	39.6	31.3	39.8	1013	46.1	58.6	1.40	
1.45	1.03			31.4	37.5	29.3	37.3	997	45.3	57.7		1
1.17	1.12			31.9	37.9	29.7	37.8	979	44.5	56.7		2
1.31	1.08	....	....	31.7	37.7	29.5	37.6	988	44.9	57.2	1.36	
....	2.46			35.9	58.8	35.0	44.6	356	44.5	56.7		2
....	3.32			35.7	58.4	35.0	44.6	360	45.0	57.2		3
....	2.89	....	....	35.8	58.6	35.0	44.6	358	44.8	57.0	1.36	
....	1.98			31.6	50.6	29.6	37.6	624	44.6	56.7		0
....	2.18			31.7	50.7	29.6	37.7	622	44.4	56.5		0
....	2.08	....	....	31.7	50.7	29.6	37.7	623	44.5	56.6	1.35	

|| No yield load indicated by load rate data.

¶ Ratio of average rivet strength in the joint to strength of undriven rivets in double shear.

** One, two, or more rivets failed, as indicated by snaps in joint during loading; number could not be observed.

varied from 3 to 6 in., in order to determine the effect of pitch on strength of joint.

The results of tests on Series A and Series B are shown in Table 9. This table also gives the test results for two pairs of specimens of Series C and D (CCC7 and DCC7), which specimens failed in the rivets.

Series A

Strength.—The average strength of the rivets at ultimate load was obtained by dividing the total load by the total shearing area of rivets, based on the nominal diameter of the rivet. As between large and small joints of the same plate and rivet steel, the maximum difference in average rivet strength was only about 10%, as shown in Table 10, composed of selected data from Table 9.

TABLE 10.—RIVET STRENGTH OF LARGE AND SMALL JOINTS; SERIES A
(Average Rivet Strength at Ultimate Load)

Plate steel	Carbon-Steel Rivets						Manganese-Steel Rivets					
	Kips per Square Inch			Percentages			Kips per Square Inch			Percentages		
	One plate	Two plates	Three plates	One plate	Two plates	Three plates	One plate	Two plates	Three plates	One plate	Two plates	Three plates
Carbon.....	55.7	51.1	50.0	100	92	90	75.9	77.4	71.4	100	102	94
Silicon.....	54.8	54.0	51.1	100	99	93	72.4	78.0	72.7	100	108	100
Nickel.....	....	....	....	....	....	....	74.7	77.7	75.7	100	104	101

For the joints that had carbon-steel rivets in combination with a given plate material, the highest rivet strength was exhibited by the short (single-plate) joints and the lowest by the long (triple-plate) joints. However, for the joints that had manganese-steel rivets, the intermediate (double-plate) joints exhibited the highest rivet strength.

In order to investigate the reason for the somewhat higher strength of the manganese-steel rivets in the double-plate joints, two selected rivets from each of three joints were subjected to hardness tests. The rivets in the double-plate joint were found to be harder than those in the single-plate and triple-plate joints; furthermore, photomicrographs showed that the rivets in the double-plate joint had a finer grain structure. The difference may have been due to variations in cooling conditions. Another variable was the driving temperature, which averaged 100° F lower for the rivets of intermediate grip than for the other rivets.

The rivet strength ratio, or the ratio of the average shearing resistance per rivet in a joint to the shearing strength of non-driven rivets in double shear, is shown in Table 5, Column (28), and is summarized in Table 11.

The manganese-steel rivets exhibited higher rivet strength ratios than the carbon-steel rivets. As between the two kinds, the actual length of splice may have had some influence upon the rivet strength ratio; the joints with carbon-steel rivets contained two more rows of rivets per lap and were somewhat

longer than the corresponding joints with manganese-steel rivets (see Figs. 2, 3, and 4; Series A). For these joints, all of which failed in the rivets, the kind of plate steel appeared to have no marked or consistent influence on the strength or the rivet strength ratio. As a group, the joints having manganese-steel rivets and nickel-steel plates exhibited the highest rivet strength ratios of any of the joints in the entire investigation.

TABLE 11.—RIVET STRENGTH RATIOS; SERIES A
(Percentages)

Plate Steel	Carbon-Steel Rivets			Manganese-Steel Rivets		
	One plate	Two plates	Three plates	One plate	Two plates	Three plates
Carbon	133	122	119	139	142	131
Silicon	130	129	121	133	143	134
Nickel	....	....	....	137	143	139

Effective Rivet Yield.—In an under-riveted joint the effective rivet yield has more significance, so far as rivet performance is concerned, than the yield strength of the joint. Average values of effective rivet yield, yield strength, and ultimate strength, taken from Table 9, are shown in Table 12. For a given

TABLE 12.—EFFECTIVE RIVET YIELD, YIELD STRENGTH, AND ULTIMATE STRENGTH; SERIES A

Plate Steel	Carbon-Steel Rivets						Manganese-Steel Rivets					
	Rivet Stress, in Kips per Square Inch			Ratios of Rivet Shears			Rivet Stress, in Kips per Square Inch			Ratios of Rivet Shears		
	Effective rivet yield	Yield	Ultimate	Column (2) / Column (3)	Column (2) / Column (4)	Column (3) / Column (4)	Effective rivet yield	Yield	Ultimate	Column (8) / Column (9)	Column (8) / Column (10)	Column (9) / Column (10)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Carbon	33.2	37.7	52.3	0.88	0.63	0.72	47.5	57.4	74.9	0.83	0.63	0.77
Silicon	32.3	40.2	53.3	0.80	0.61	0.76	46.1	60.1	74.4	0.77	0.62	0.81
Nickel	....	....	....	....	....	....	46.0	66.0	76.0	0.70	0.61	0.87
Average	32.8	39.0	52.8	0.84	0.62	0.74	46.5	61.2	75.1	0.76	0.62	0.82

rivet steel, the effective rivet yield did not vary greatly; and for both kinds of rivet steel, the effective rivet yield averaged 62% of the ultimate strength of rivets in the joints of this series (Series A). The yield strength of the joints varied with the type of plate steel as well as the type of rivet steel, so that the ratio of yield strength to ultimate strength was not constant.

The ratio of effective rivet yield to the yield strength and ultimate strength in tension of rivet-rod stock did not differ greatly as between the two kinds of

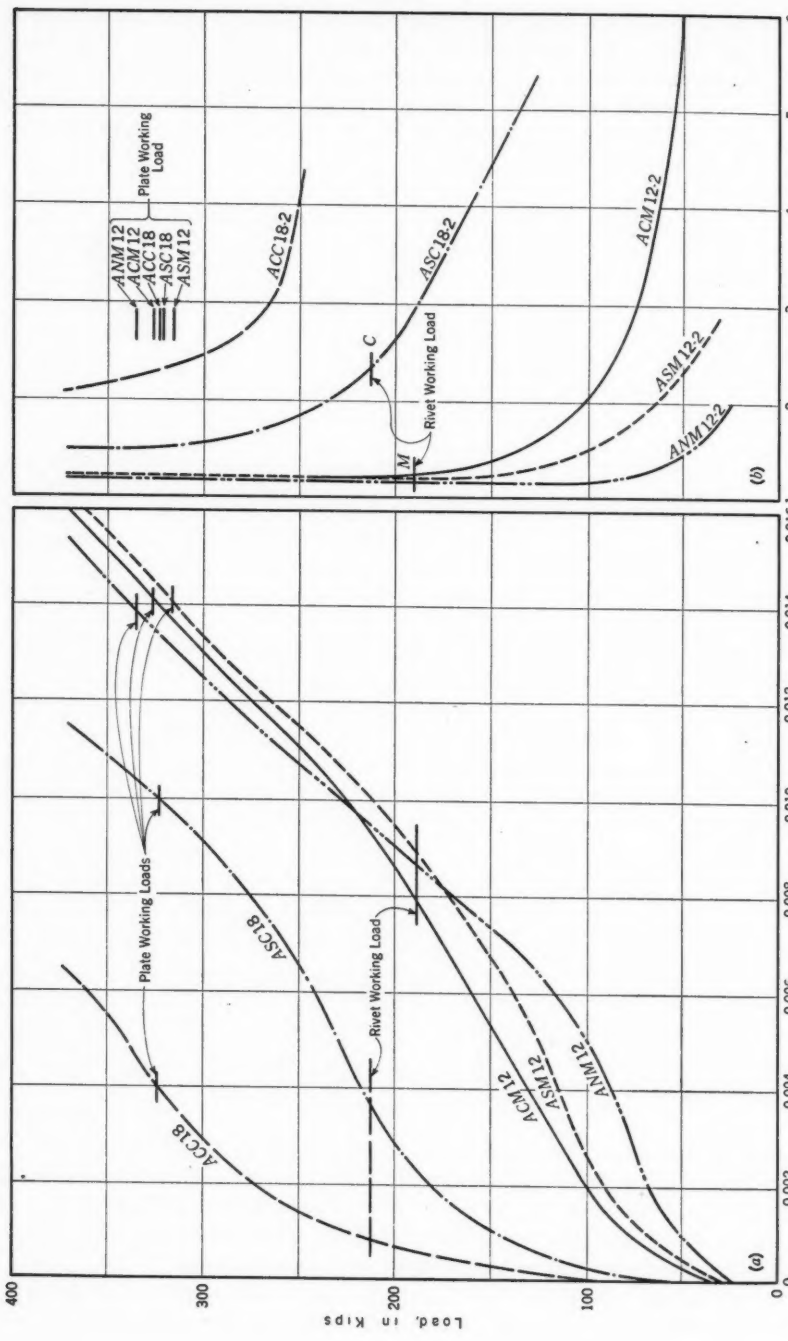


FIG. 8.—LOAD-SLIP RELATIONS FOR SINGLE-PLATE JOINTS OF SERIES A

rivet steel; for example:

Ratio	Carbon-steel rivets	Manganese-steel rivets
$\frac{\text{Effective rivet yield}}{\text{Yield strength in tension}}$	0.83	0.85
$\frac{\text{Effective rivet yield}}{\text{Tensile strength}}$	0.57	0.57

Slip.—The results of observations to determine slip for the joints of Series A are shown in Figs. 8 to 10. For the single-plate joints, Fig. 8(*a*) shows the

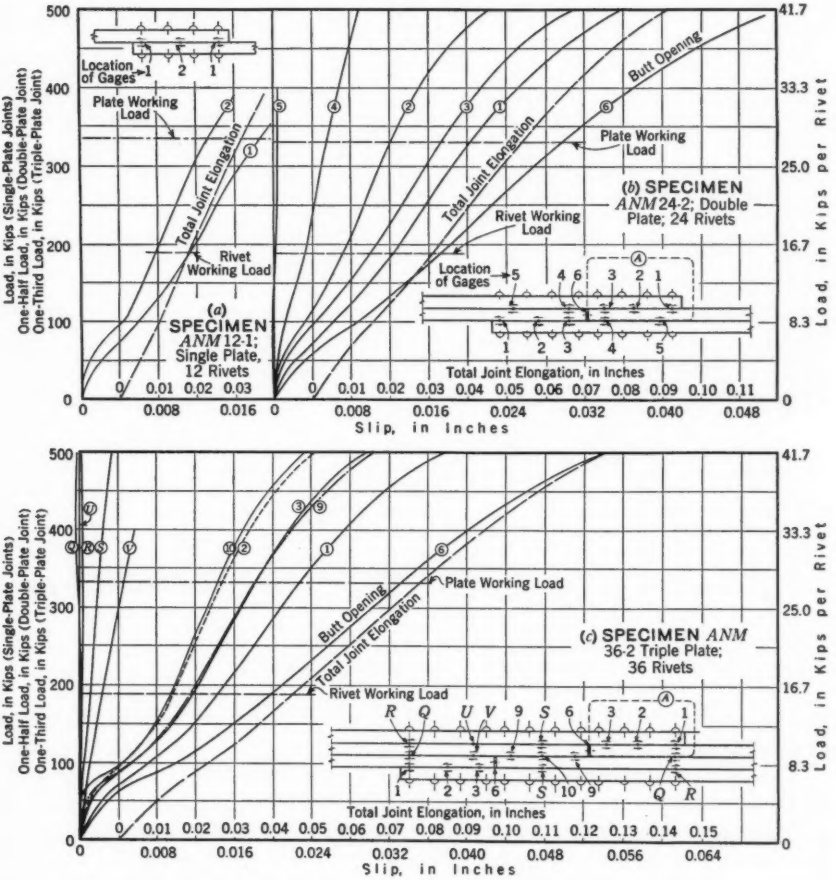

Load, in Kips (Single-Plate Joints)
One-Half Load, in Kips (Double-Plate Joint)
One-Third Load, in Kips (Triple-Plate Joint)
Load, in Kips per Rivet
Slip, in Inches
Location of Gages
Plate Working Load
Total Joint Elongation
Rivet Working Load
Butt Opening
(a) SPECIMEN ANM 12-1; Single Plate, 12 Rivets
(b) SPECIMEN ANM 24-2; Double Plate; 24 Rivets
(c) SPECIMEN ANM 36-2 Triple Plate; 36 Rivets
Total Joint Elongation, in Inches

FIG. 9.—LOAD-SLIP RELATIONS AT VARIOUS PLACES ON TYPICAL SPECIMENS OF SERIES A

relation between load and middle slip. The joints with eighteen carbon-steel rivets (Specimens ACC18 and ASC18) slipped considerably less than those with twelve manganese-steel rivets, even if the slip were taken at the same load per rivet. Of the joints with carbon-steel rivets, those having carbon-steel

plates slipped less than those having silicon-steel plates. All of the joints having manganese-steel rivets exhibited similar slip characteristics.

Fig. 8(*b*) shows the relation between load and the ratio of end slip to middle slip for one specimen of each of the pairs of single-plate joints. The end slip was measured midway between the first and second rows of rivets. Values in Fig. 8(*b*) are for one specimen of each pair. A constant ratio of end slip to middle slip—taken to be an indication that the rivets had come into full bearing—was attained at lower loads by the joints with twelve manganese-steel rivets than by those with eighteen carbon-steel rivets. Probably the difference

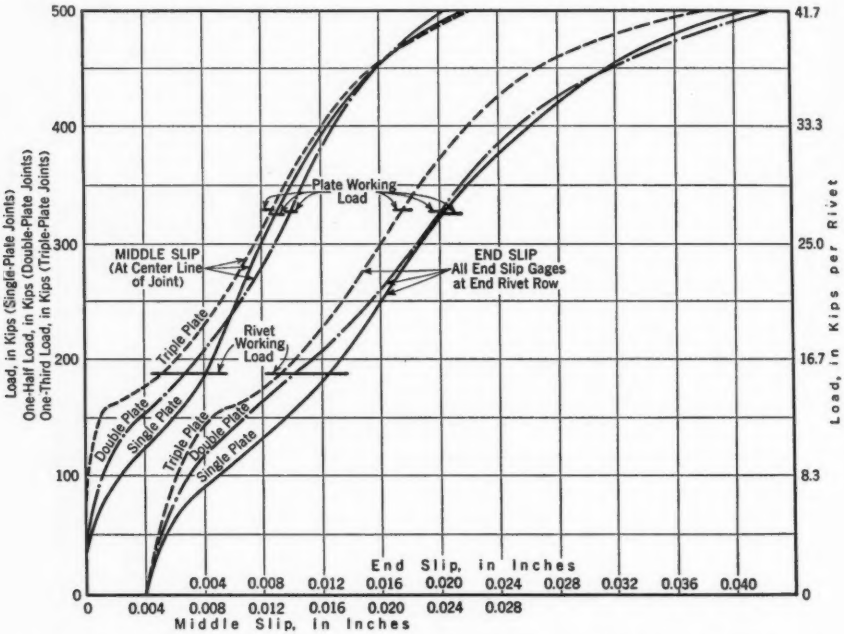

Load, in Kips (Single-Plate Joints)
One-Half Load, in Kips (Double-Plate Joints)
One-Third Load, in Kips (Triple-Plate Joints)
Load, in Kips per Rivet
Plate Working Load
MIDDLE SLIP (At Center Line of Joint)
END SLIP All End Slip Gages at End Rivet Row
Rivet Working Load
Triple Plate
Double Plate
Single Plate
Middle Slip, in Inches
End Slip, in Inches

FIG. 10.—LOAD-SLIP RELATIONS; TYPICAL SINGLE-PLATE, DOUBLE-PLATE, AND TRIPLE-PLATE SPECIMENS OF SERIES A (SPECIMENS ACM)

was due largely to the difference in number of rows of rivets and in characteristics of the two rivet materials.

The slip characteristics exhibited by the double-plate and triple-plate joints were similar to those exhibited by the single-plate joints. As representative of the behavior of the series, the relation between load and the slip measured at various points along joints of the ANM group is shown in Fig. 9.

The plates were all of nickel steel, $\frac{5}{8}$ in. by 18 in.; and all rivets were of manganese steel, 1 in. in diameter. The relative movement of plates at the slip-gage locations is indicated by half-arrows in the inset diagrams. In the double-plate joint, the portion enclosed by the dotted line marked "A" (see inset, Fig. 9(*b*)) may be considered the equivalent of a single-plate lap joint, since at the end of the double-plate joint half of the total load is carried by

each plate; and, at the middle of length of the joint, the load on the inner plate is zero while the outer plate still carries half the load. Similar characteristics hold for the triple-plate joint. Comparison of the end slips at Gage Locations 1 of the single-plate joint with those at Gages 1 and 3 of the double-plate joint, and at Gages 1, 3, and 9 of the triple-plate joint, shows that the slips at both ends of equivalent single laps in the multiple-plate joints did not differ greatly from the end slips for the single-plate joint at corresponding loads. Furthermore, in the multiple-plate joints, the "middle" slips (measured at all Gage Locations 2 and, for the triple-plate joint, at Gage 10 also) were similar to the middle slip of the single-plate joint. Slips at other points show the relative movements of the plates throughout the length of the joints.

A comparison of the relative magnitude of end and middle slip for a single, a double, and a triple-plate joint is afforded by the curves in Fig. 10. For the same unit stress in the plates, the middle slips and likewise the end slips were about the same as between the three sizes of joint. As in Fig. 8(*b*), the values are for one specimen of each pair.

The values of end slip at the rivet working load are summarized in Table 9, Column (16). With a few individual exceptions, the slips for single, double, or triple plates in any group having the same plate and rivet material were about the same. As noted in connection with the single-plate joints, the joints having carbon-steel rivets exhibited less slip than corresponding specimens having two-thirds as many manganese-steel rivets.

Four stages of action in riveted joints are indicated by the load-slip relations (see Fig. 1 and accompanying text). For the single-plate joints of Series A,

TABLE 13.—AVERAGE STRESS IN RIVETS AT VARIOUS STAGES OF JOINT ACTION; SERIES A

Plate steel	Eighteen Carbon-Steel Rivets					Twelve Manganese-Steel Rivets				
	Kips per Square Inch					Kips per Square Inch				
	At working load	At end of Stage I	At beginning of Stage III	At effective rivet yield (in Stage IV)	Ratio, Column (4) / Column (2)	At working load	At end of Stage I	At beginning of Stage III	At effective rivet yield (in Stage IV)	Ratio, Column (9) / Column (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Carbon....	15	6	27	33	1.8	20	5	23	47	1.2
Silicon.....	15	4	21	32	1.4	20	3	19	46	0.9
Nickel.....	....	....	....	....	....	20	2	13	46	0.7

Table 13 shows the approximate loads at the end of Stage I (no slip at middle of joint), at the beginning of Stage III (full bearing of all rivets, linear load-slip relation), and at the effective rivet yield which occurs when Stage IV is well started. For most of the joints, the rivet working load fell in Stage II; only for the specimens with manganese-steel rivets and nickel-steel plates was the working load definitely in Stage III.

Elongation.—The unit elongation of each joint relative to that of the main plate (gross section) varied from 1 to 2 at the plate working load (Table 9, Column (18)). The larger the joint, the less was the elongation ratio. It appears that the ratio was affected principally by the factors that influenced slip. For a single, a double, and a triple-plate specimen having nickel-steel plates and manganese-steel rivets, the variation in total elongation with increase in load is shown in Fig. 9.

Distribution of Strain.—The variation in longitudinal strain along a single, a double, and a triple-plate joint is illustrated in Fig. 11, for a stress at net section of 20 kips per sq inch. These curves show that in the multiple-plate joints the stress in the outer plates was maintained at approximately the full value up to the beginning of the lap at the far end of the joint (marked "X"); between this point and the end of the plate the decrease in stress was similar to that in a single-plate lap joint. Irregularities in the strain diagrams may be due in part to localized stresses near the rivets and inequalities arising from fabrication.

Typical variations in longitudinal strain across a carbon-steel plate ($\frac{5}{8}$ in. by 26 in.) are shown in Fig. 12. The diagrams for Specimen ACC18-2 (eighteen, 1-in. carbon-steel rivets) show the observed strains, the corrections in strain determined by means of the curvemeter, and the corrected strains, for a plate stress (at net section) of 20 kips per sq in. The diagrams for Specimen ACM12-2 (twelve, 1-in. manganese-steel rivets) show the corrected strains at plate stresses of 10 and 20 kips per sq in. The average strain across any section, employed in the load-partition studies discussed subsequently, was obtained by measuring, by planimeter, the areas under the corrected strain-distribution curves for each load.

Partition of Load Among Rivets.—The partition of load among rivets in the various transverse rows, determined from strain measurements along gage lines located between the rivet rows, is shown in Fig. 13. Data are shown for selected single-plate joints of each of the five types; for all of these specimens the bending frame (Fig. 5) was used to reduce the curvature of the plates, and curvemeter observations were made to provide a correction for change in curvature during loading. The curves show the percentage of the total load taken by a rivet in each row. Although there may be considerable error in the individual results, it is believed that the values are sufficiently accurate to show quantitatively the action of the joints tested. For this series, the specimen number defines the physical characteristics of each specimen; thus: ACM12-2 denotes Series A; Carbon-steel plates; Manganese-steel rivets; 12 rivets per joint; and the second specimen of the pair tested.

The load partition is shown at each of four loads, which differ as between the several types of joint. These loads varied from about one-half to about twice the rivet working loads. All the loads at which observations were taken lay above Stage I of joint action. Most of the observations were taken at loads within Stage II, which ended at rivet stresses of about 13 to 27 kips per sq in. For the joints with manganese-steel rivets, the third and fourth loads lay in Stage III; and for the joints in this group which had nickel-steel plates, the second load also lay in Stage III.

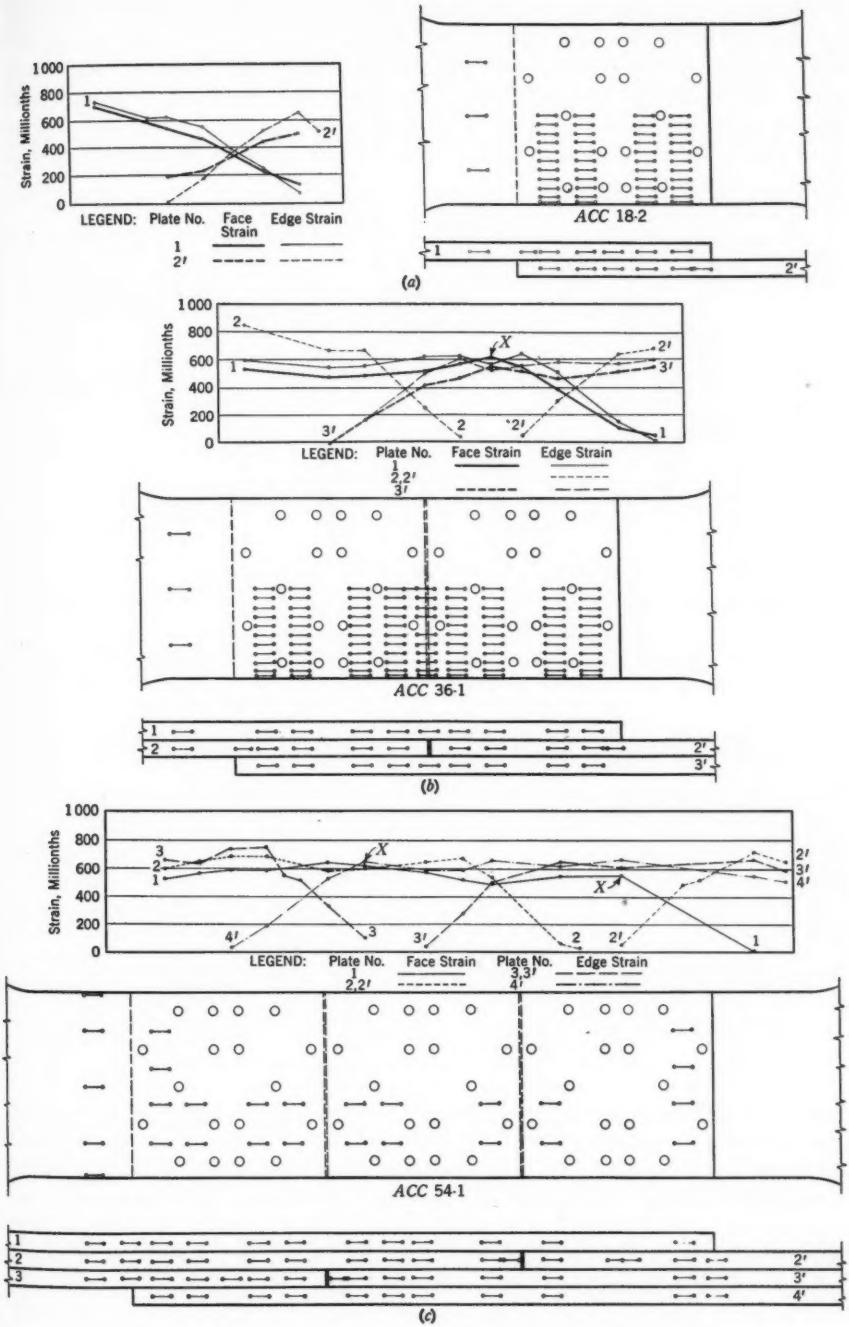


FIG. 11.—VARIATION IN STRAIN ALONG TYPICAL JOINTS OF SERIES A

In some cases there was a markedly greater proportion of load transferred at the end rows of rivets than at the interior rows, and to this extent the tendency of the joints to behave in accordance with elastic theory[9] was borne out. For example, in Joint ACC18-2, which had six rows of carbon-steel rivets, the load taken by a rivet in the end rows was about twice the average load per rivet. In Joint ANM12-1, which had four rows of manganese-steel

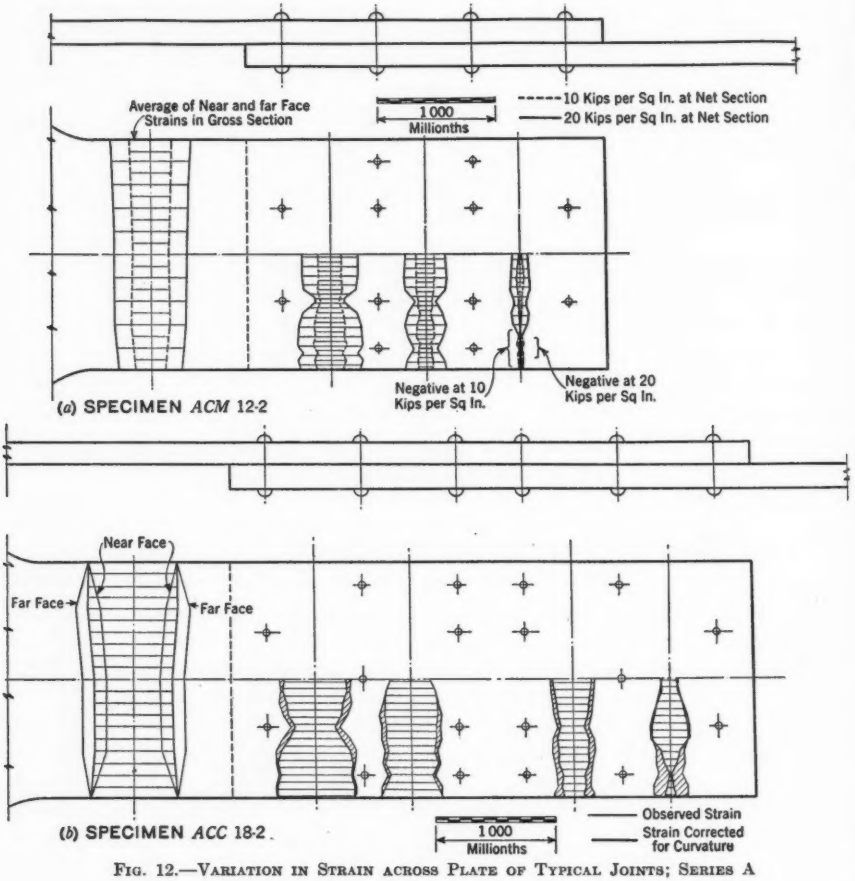

Average of Near and far Face Strains in Gross Section
1 000 Millionths
10 Kips per Sq In. at Net Section
20 Kips per Sq In. at Net Section
Negative at 10 Kips per Sq In.
Negative at 20 Kips per Sq In.
(a) SPECIMEN ACM 12-2
Near Face
Far Face
Far Face
(b) SPECIMEN ACC 18-2
1 000 Millionths
Observed Strain
Strain Corrected for Curvature

FIG. 12.—VARIATION IN STRAIN ACROSS PLATE OF TYPICAL JOINTS; SERIES A

rivets, the load taken by a rivet in the end rows was about 20% higher than the average load per rivet. As a whole, however, the observed partitions of load among rivets cannot be said to conform to those that would be computed by the elastic theory. The effect of imperfect fabrication, as indicated by irregularities in load partition, is very apparent and may be as large as the difference between uniform load partition and that which might be expected in a joint that is assumed to behave elastically.

For joints that exhibited relatively high stresses on end rivets and for which two or more of the loads lay in Stage II (ACC18-2, for example) increasing the

load reduced the share taken by the end rivets and tended toward more uniform partition of load. Upon this observation is based the preceding general statement that load partition is more nearly uniform in Stage II than in Stage I in which the joint may be behaving elastically (see text applying to Fig. 1).

For Specimen ANM12-1, in which the load partition tended to approach that required by elastic theory from the beginning, the relative stresses taken

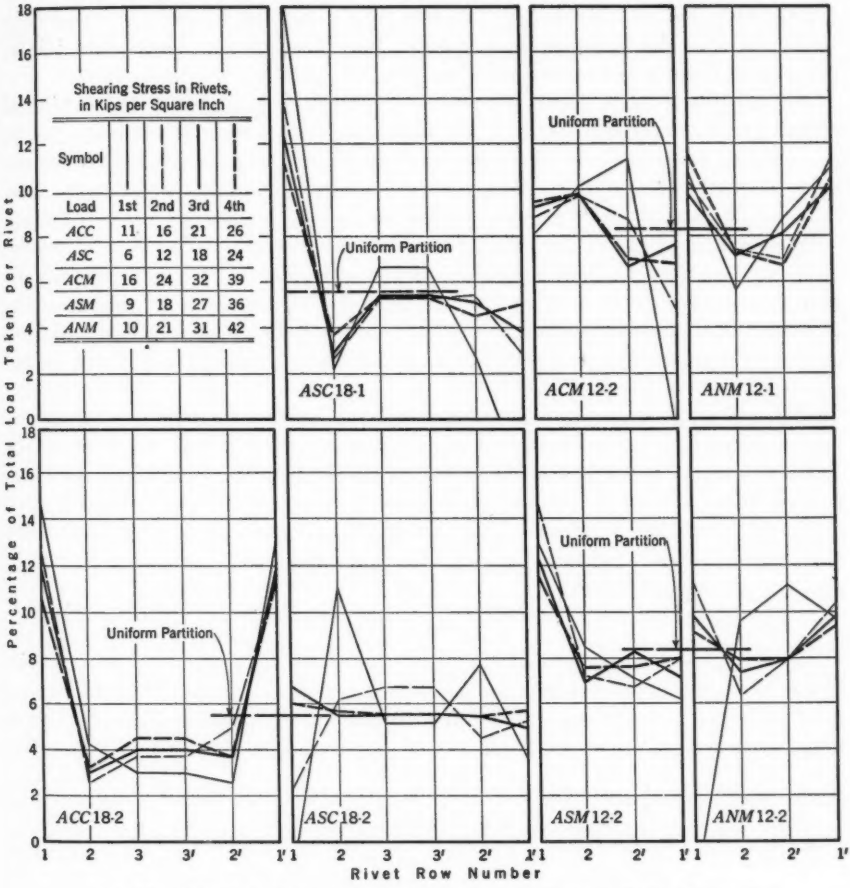


FIG. 13.—PARTITION OF LOAD AMONG RIVETS IN THE VARIOUS ROWS; SERIES A

by the end rivets increased for the last two loads which were definitely in Stage III. This would indicate that after Stage II is passed, the load partition is again built up so as to approach, but not necessarily to conform to, that indicated by the elastic theory.

The elastic theory is not considered to be suitable for the prediction of load partition among rivets of a joint because: (1) The partition of load tends to vary through the successive stages of joint action, (2) ideal joints are not produced

commercially, and (3) considerable slip occurs within the working range so that—except for short joints with high-strength plate and rivet materials—the rivets do not come into full bearing until the joint is stressed above the working load.

Premature Rivet Failures.—The number of individual rivets failing before complete rupture of the joint is shown in Table 9, Column (29). In nearly all cases of premature rivet failure, the rivets were in the end row of a lap; in the remaining cases, the rivets were in the second row of a lap and failed only after rivets in the first row had failed. For three of the triple-plate joints, premature failures occurred in the end row of rivets of an interior lap.

Of the single-plate joints of Series A, only one pair (ACC-18) exhibited premature rivet failure (Table 9, Column (29)). In the multiple-plate joints, the number of premature rivet failures was: (*a*) Greater for triple-plate joints than for double-plate joints; (*b*) greater for joints with carbon-steel rivets than for joints with two-thirds as many manganese-steel rivets; (*c*) greater for joints with carbon-steel plates than for joints with silicon-steel or nickel-steel plates; and (*d*) zero for the joints having nickel-steel plates in combination with manganese-steel rivets. It may be concluded that the strain imposed on the end rivets at loads approaching the ultimate for the joint is greater for the larger joints, greater for the longer (less compact) joints of a given nominal size, and greater for the more ductile steels.

The occurrence of premature rivet failures at the ends of a joint is not necessarily an indication that at loads approaching failure the end rivets carry greater than the average load per rivet. Although direct evidence is lacking, it seems likely that the load partition between rivets tends toward uniformity while the rivets are yielding. The fact that premature rivet failures did not occur with the less ductile steels indicates that excessive detrusion, rather than stress, caused such failures.

Series B

In the specimens of Series B (Fig. 2), the rivets were arranged in full rows of 4 each, at a gage distance of 7.5 in., which is wider than would ordinarily be used with full-row riveting. The approximate values of load at various stages of joint action are shown in Table 14. Compared with similar joints in Series A, the joints of Series B exhibited relatively low slip. For example, as shown by the values in Table 9, Column (16), the slip of the joints of Series B at the rivet working load was less than half that of Joints ACC18.

TABLE 14.—AVERAGE STRESS IN RIVETS AT VARIOUS STAGES OF JOINT ACTION; SERIES B

Specimens	Rivet Stress, Kips per Square Inch		
	Working Load	End Stage I	Begin Stage III
BCC20*a*	15	10	25
BCC20*b*	15	9	23
BCC20*c*	15	7	21

The differences in strength for all the specimens of Series B were small, although the joints with a pitch of 4.5 in. (0.6 of gage distance) were somewhat stronger than either the shorter or the longer joints. If the gage distance had been smaller (say, four or five rivet diameters) the optimum ratio of pitch to gage would not necessarily have been the same as in these specimens. Test results are summarized in Table 15. Specimens ACC18 of Series A (see

TABLE 15.—EFFECT OF PITCH ON STRENGTH; SERIES B

Series	Number of rivets in end row	Pitch, in inches	Average Shear in Rivets at Ultimate Load		Number of premature rivet failures* (All in end rows)
			Kips per square inch	Percentages	
B	4	3	56.4	96	0
		4.50	58.6	100	1
		6	57.2	98	3
A	2	5.25	55.7	95	2 or more

* In two specimens.

Fig. 2) were not greatly different from the specimens of Series B except with regard to rivet pattern; in specimens ACC18 the net section was supposedly protected by having only two rivets in the first row and three rivets in the second row. Table 15 shows that these joints were not quite as efficient as the joints of Series B which had the full number of rivets in the end row. The last column of Table 15 confirms the discussion of Series A to the effect that end rows of rivets are subjected to larger strains in long splices than in short splices.

TESTS OF BUTT AND SHINGLE JOINTS; SERIES F

For heavy tension members, the use of a "shingle" splice results in considerable economy of material and a shorter rivet grip than does the conventional butt splice. The specimens of Series F, shown in Fig. 4, were designed for comparison between butt and shingle splices. The plates were of silicon steel, 20 in. wide, and all main plates were $\frac{11}{16}$ in. thick. The rivets were of carbon steel and were $\frac{7}{8}$ in. in diameter. The distinguishing characteristics of the six pairs of specimens were as shown in Table 16. With the intention

TABLE 16.—TYPES OF SPECIMEN; SERIES F

Number of main plates	Symbol	Type of splice	Number of rivets
1	FSCA	Butt	38
2	FSCB	Butt	76
	FSCD	Shingle	76
3	FSCC	Butt	114
	FSCE	Shingle	114
	FSCF	Half-shingle	85 (116 shearing surfaces)

of insuring plate failure, the ratio of tensile stress in the plates to shearing stress in the rivets (T : S ratio) was made 1 : 0.54 (28 000 : 15 000) instead of the

1 : 0.57 (93 000 : 53 000) established for rivet failure as a result of the pilot tests. In spite of this provision, in all specimens, except the single-plate butt joints FSCA, one or more rivets failed before plate failure occurred. The results of the tests are shown in Table 17 and Figs. 14 to 18 inclusive.

Behavior of Individual Joints

In order to secure a more general picture of the behavior of the parts of these joints than that afforded by the slip measurements alone, a series of

TABLE 17.—TESTS ON BUTT

Specimen No. (for details, see Fig. 4).	Type of joint	Length of joint, in inches*	Plates (11/16 in. by 20 in.; Silicon Steel)			Rivets		Mill Coupons (Kips per Square Inch)		Specimens
			Number of main plates	Area, in Square Inches		Total number	Number of shearing surfaces	Yield strength	Tensile strength	End slip at rivet working load, in inches
				Gross	Net (first row)					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
FSCA-1				13.74	12.37			51.7	92.1	0.0078
FSCA-2	Butt	33½	1	13.84	12.46	38	38	51.3	90.8	0.0064
Average				13.79	12.42			51.5	91.5	0.0071
FSCB-1				27.58	24.82			51.2	90.5	0.0056
FSCB-2	Butt	63½	2	27.62	24.86	76	76	52.8	93.3	0.0038
Average				27.60	24.84			52.0	91.9	0.0047
FSCD-1				27.51	24.76			50.9	90.1	0.0046
FSCD-2	Shingle	63½	2	27.66	24.89	76	76	51.7	90.6	0.0046
Average				27.59	24.83			51.3	90.4	0.0046
FSCC-1				41.19	37.07			54.0	92.5	0.0077
FSCC-2	Butt	87½	3	41.00	36.90	114	114	53.7	90.8	0.0075
Average				41.10	36.99			53.9	91.8	0.0076
FSCE-1				40.16	36.15			52.5	91.9	0.0057
FSCE-2	Shingle	87½	3	41.03	36.93	114	114	52.6	93.1	0.0061
Average				40.60	36.54			52.6	92.5	0.0059
FSCF-1	Half-			40.96	36.87			51.5	90.4	0.0052
FSCF-2	Shingle	66½	3	41.26	37.13	85	116	51.4	90.2	0.0056
Average				41.11	37.00			51.5	90.3	0.0054

* Length of inner (long) splice plates.
† Ratio of average unit elongation of joint to unit elongation of plates (gross section).
‡ Ratio of stress in gross section of plate to corresponding strength of mill coupons.

2.5-in. strain-gage lines was located on the edges of the plates, for one specimen of each pair. In calculating the share of the load taken by the various plates at any given section from the edge strains, it was necessarily assumed that the load transferred across a section was proportional to the observed strain. It is believed that the error involved in this assumption was reasonably small in the case of the multiple-plate joints since these joints had full-row riveting over a large proportion of the length.

The behavior under load and the manner of failure, which are of particular interest in the case of the multiple-plate joints, are shown in Figs. 14 to 17, inclusive. In the figures, Diagrams *A* and *B* are the edge and face views of a joint and show rivet locations, gage-line locations, and the lines along which the parts of the joint separated at failure. In general, the location of final

failure is shown for one of the two specimens at the left end of the diagrams and for the other specimen at the right end. In Diagrams *B*, a cross or a heavy dot drawn at a rivet location indicates that the rivet failed before complete rupture of the joint. Diagrams *C* show the percentage of load taken by each plate; the values were obtained from strain data by taking the summation of strains in all plates at any transverse section as 100 per cent. The stepped lines show the load that would be taken by each plate, calculated on the assumption of equal partition of load among rivets.

AND SHINGLE JOINTS; SERIES F

SPECIMENS (*Continued*)

Elongation ratio at plate working load†	At Yield Load				At Ultimate Load						
	Stress in Plate, in Kips per Square Inch		Efficiency, percentages‡	Shear in rivets, in kips per square inch	Location of failure§	Load, in kips	Stress in Plate, in Kips per Square Inch		Efficiency, percentages‡	Shear in Rivets, in Kips per Square Inch	
	Gross	Net					Gross	Net		All rivets‖	Remaining rivets¶
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
1.13	45.3	50.3		27.2	P	941	68.5	76.1		41.2	41.2
0.95	44.2	49.1		26.8	P	957	69.1	76.8		41.9	41.9
1.06	44.8	49.7	87.0	27.0		949	68.8	76.5	75.2	41.6	41.6
0.96	44.0	48.9		26.6	PR	1855	67.3	74.7		40.6	42.8
0.90	42.2	47.0		25.5	PR	1845	66.8	74.2		40.4	42.6
0.93	43.1	48.0	82.9	26.1		1850	67.1	74.5	73.0	40.5	42.7
0.80	42.4	47.2		25.5	PR	1782	64.8	72.0		39.0	40.6
0.85	43.7	48.6		26.5	PR	1816	65.7	73.0		39.7	41.9
0.83	43.1	47.9	84.0	26.0		1799	65.2	72.5	72.1	39.4	41.3
0.81	45.9	51.0		27.6	PR	2694	65.4	72.7		39.3	41.5
0.86	44.5	49.5		26.6	R	2626	64.0	71.2		38.3	46.5
0.84	45.2	50.3	83.8	27.1		2660	64.7	72.0	70.5	38.8	44.0
0.79	45.1	50.1		26.4	PR	2702	67.3	74.7		39.4	42.4
0.83	42.4	47.1		25.4	PR	2674	65.2	72.4		39.0	42.8
0.81	43.8	48.6	83.3	25.9		2688	66.2	73.6	71.6	39.2	42.6
0.90	43.6	48.4		25.6	PR	2595	63.4	70.4		37.2	37.9
0.93	42.7	47.4		25.2	PR	2708	65.6	72.9		38.8	39.9
0.92	43.2	47.9	83.9	25.4		2652	64.5	71.7	71.4	38.0	38.9

§ P = plate; and R = rivet.
‖ Based on total number of rivets in joint.
¶ Excluding rivets which failed before maximum load was attained.

In all the specimens of this group of tests (Figs. 14 to 17) the main plates were $\frac{11}{16}$-in. by 20-in. silicon-steel plates, held by $\frac{7}{8}$-in. carbon-steel rivets (see Fig. 4). Slip gages were attached to both edges. In the single-plate and double-plate joints (Figs. 14 and 16(*a*)) the stress at the net section, 18.75 kips per sq in., was the average of 7.5, 15, 22.5 and 30 kips per sq in. In the triple-plate joints (Figs. 15, 16(*b*), and 17) the stress at the net section, 22.5 kips per sq in., was the average of 15 and 30 kips per sq in.

Single-Plate Butt Joints FSCA.—The strains in the main and splice plates were measured along chains of 2.5-in. gage lines, as shown in Fig. 14(*a*), Diagram *A*. The load partition computed from the strain is shown in Diagram *C*. Studies based on these data indicate that at the given loads the rivets in the end rows took more than the average share of the load, although the observed

strains were undoubtedly affected by their location with respect to the various rivet rows which contained different numbers of rivets. The specimens failed entirely in the plates through an end row of rivets, as shown in Diagrams *A* and *B*, Fig. 14(*a*). First, the plate was torn from one end rivet to the edge, then the entire plate ruptured suddenly. The fracture was square and of a coarse, granular texture. Scaling of brittle paint from rivet heads occurred only in the end rows.

Double-Plate Butt Joints FSCB.—Strains were measured along chains of 2.5-in. gage lines on the edges of the plates, as shown in Fig. 14(*b*), Diagram *A*. Studies indicated that over the given range of loading the load per rivet was

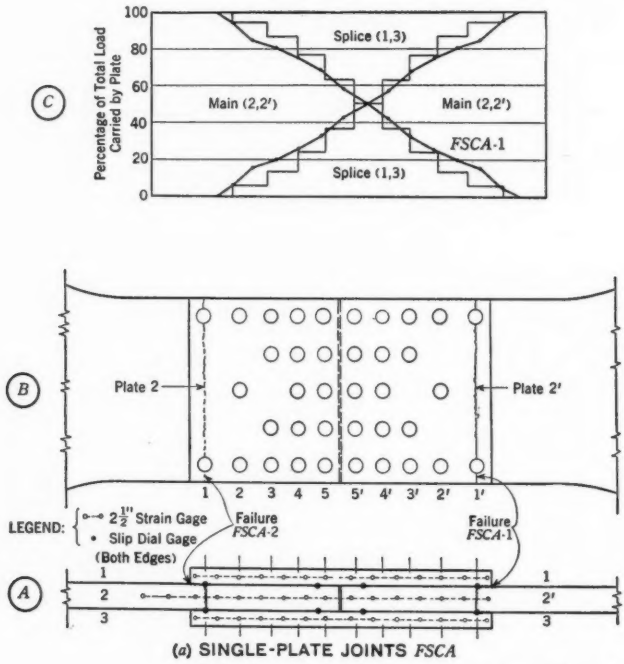


FIG. 14.—PARTITION OF LOAD AMONG PLATES,

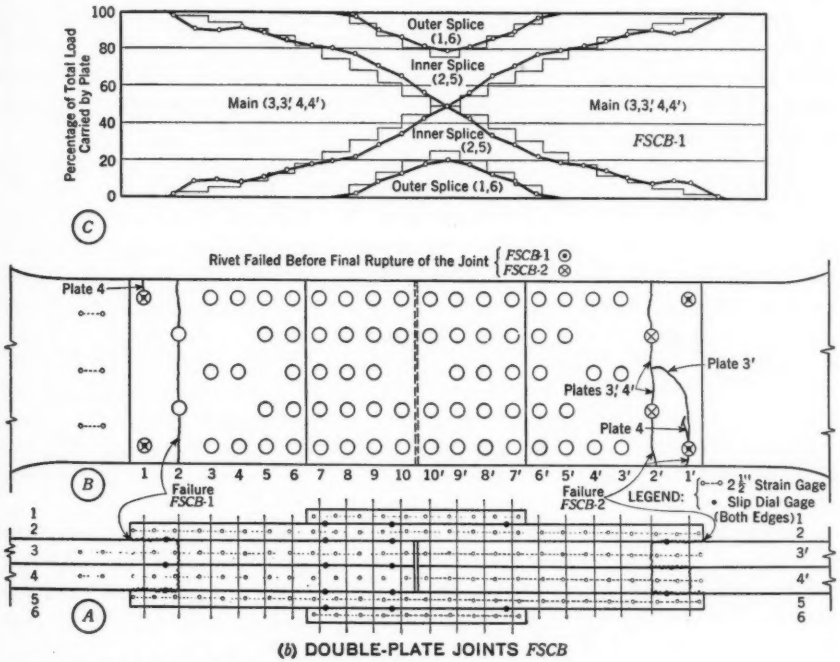
highest in the end row and in the two interior rows (9 and 10) next to the butt. In general, the progress of rivet scaling confirmed the load distribution determined from the strain measurements; marked scaling occurred in Rows 1, 10, 9, 8, and 7 in that order, whereas there was little scaling in Rows 2 and 3 and none in Rows 4, 5, and 6. At the end of the outer (short) splice plates, the inner (long) splice plates carried about 41% of the total load, as shown in Diagram *C*, Fig. 14(*b*). At the butt, the inner splice plates comprising 54% of the splice material carried about 58% of the total load, and the outer splice plates comprising 46% of the splice material carried about 42% of the load.

Before final rupture of the joint as a whole, all end-row rivets in both specimens failed. The plates fractured at the second row of rivets, except that one

plate of Specimen FSCB-2 fractured half across the second row and half across the end row (Diagram *B*, Fig. 14(*b*)). About 80% of the fracture was square, and of coarse, granular texture; the remainder was of fine granular to silky texture, in part of the cup-cone and in part of the angular-shear type.

The relative displacements (slip) of the plates, observed after failure by measurements between lines scribed across the edges of the plates, disclosed that at sections where the rivets had failed, before complete failure of the joint, the slip was in excess of 0.33 in.

Double-Plate Shingle Joints FSCD.—Strains were measured along chains of 2.5-in. gage lines on the edges of the plates, as shown in Fig. 16(*a*), Diagram *A*.



(*b*) DOUBLE-PLATE JOINTS *FSCB*

AND LOCATION OF FRACTURE; BUTT JOINTS

Over the stated range of loads, the load per rivet was found to be greatest in the two end rows and between the main plates at a butt. The load per rivet transferred directly from main plate to main plate at the middle of the joint was about the same as the average load per rivet. In general, the strain observations were confirmed by the progress of rivet scaling. Marked scaling occurred in Rows 1, 10, and 2 (in that order); little scaling occurred in Rows 4 to 7.

Before the joint failed as a whole, all but one of the end-row rivets failed. The plate fractures followed irregular lines which passed through rivet holes in one or more of the three rows next to an end (Diagram *B*, Fig. 16(*a*)). The fractures were in part square, with coarse, granular texture, and in part of the

cup-cone and angular-shear type, ranging from fine granular to silky in texture. Examination of the joints after fracture disclosed that wherever rivets had sheared, the relative displacement of the plates was in excess of 0.30 in.

Triple-Plate Butt Joints FSCC.—For this type of specimen, strains were measured only at selected locations along the joint, as shown in Fig. 15, Diagram *A*. The behavior of this joint was similar to that of the double-plate butt joint FSCB, in that the load per rivet was high at the ends and near the butt, and low just outside the ends of the outer (short) splice plates. In this pair of triple-plate specimens the outer splice plate was shorter than intended in the design. The progress of rivet scaling was in agreement with the computed distribution of load between rivet rows.

Along either half of the length of the joint, the twenty-seven rivets (47% of the total number) that passed through both splice plates transferred about 61% of the load from main plates to splice plates whereas the thirty rivets that passed only through the inner (long) splice plates transferred only about 39% of the load (Diagram *C*, Fig. 15).

The relatively high stress in the rivets that passed through both splice plates was reflected in the failure of Specimen FSCC-1, which failed partly in the plates and partly in the rivets, as shown in Diagrams *A* and *B*. The rivets in the end rows failed before the ultimate load was reached, and a rivet near the butt sheared at the ultimate load (Diagram *B*, Fig. 15). About 65% of the plate fracture was square, with coarse, granular texture, and the remainder was of the angular-shear type with silky texture.

Specimen FSCC-2 failed in the rivets along lines shown in Diagram *A*, although at Rivet Row 3′ (Diagram *B*, Fig. 15) a tear 4 in. long extended through a rivet to the edge in one outer main plate and a tear 0.5 in. long extended from the same rivet in the other outer main plate. Premature rivet failures occurred in five rows at each end and in two rows next to the butt (Diagram *B*).

For Specimen FSCC-1, it was found by measuring the displacement of the scribe lines which had been ruled across the edges of the joint that wherever the rivets near the end had failed the final slip was in excess of 0.33 in., and wherever rivets near the butt had failed the slip was in excess of 0.26 in. For Specimen FSCC-2, the corresponding values were 0.37 and 0.24 in.

Triple-Plate Shingle Joints FSCE.—Strains were measured at selected locations along the edges of the plates, as shown in Fig. 17, Diagram *A*. It was found that the rivets in the end rows took considerably more than the average share of the load, over the load range at which the measurements were made. The progress of rivet scaling was in agreement with the computed distribution of load between rivet rows. Between Rows 1 and 9 the scaling on the face of Plate 1 was much more severe than on the face of Plate 5, and between Rows 1′ and 9′ the scaling was much more severe on the face of Plate 5 than on the face of Plate 1.

The specimens failed partly in the plates and partly in the rivets, as shown in Diagrams *A* and *B*, Fig. 17. The rivets in the two end rows failed before the ultimate load was reached; and some rivets in the third and fourth rows sheared before final failure occurred (Diagram *B*). The plate fractures

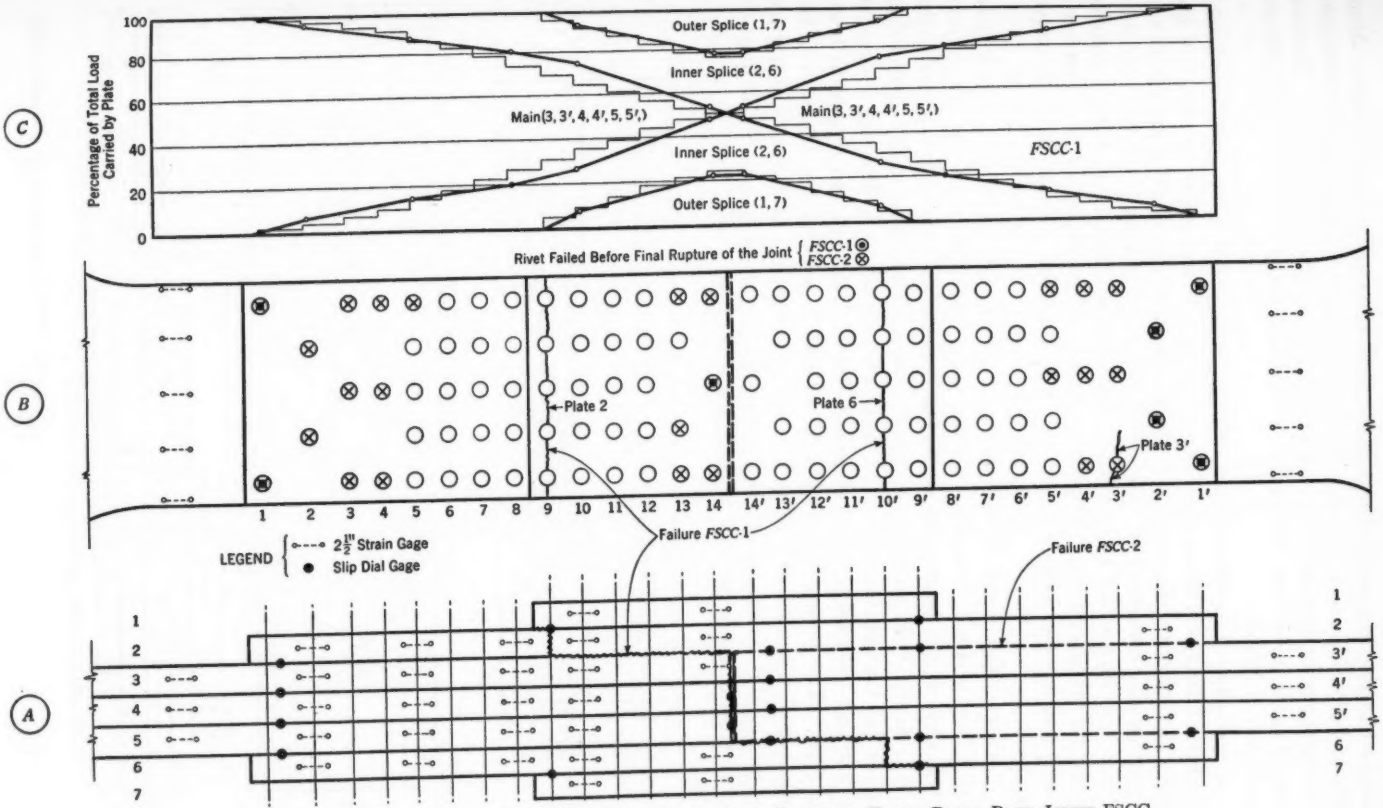
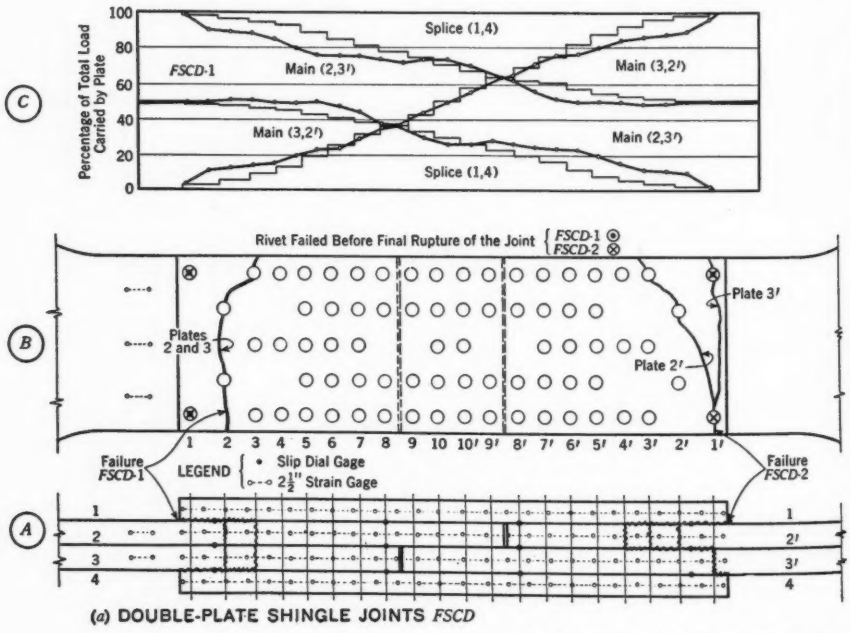

C
Percentage of Total Load Carried by Plate
100
80
60
40
20
0
Outer Splice (1, 7)
Inner Splice (2, 6)
Main(3, 3′, 4, 4′, 5, 5′,)
Main(3, 3′, 4, 4′, 5, 5′,)
Inner Splice (2, 6)
Outer Splice (1, 7)
FSCC-1
Rivet Failed Before Final Rupture of the Joint { FSCC-1 FSCC-2
B
Plate 2
Plate 6
Plate 3′
1 2 3 4 5 6 7 8 9 10 11 12 13 14 14′ 13′ 12′ 11′ 10′ 9′ 8′ 7′ 6′ 5′ 4′ 3′ 2′ 1′
Failure FSCC-1
Failure FSCC-2
LEGEND { 2½" Strain Gage
Slip Dial Gage
A
1 2 3 4 5 6 7
1 2 3′ 4′ 5′ 6 7

FIG. 15.—PARTITION OF LOAD AMONG PLATES, AND LOCATION OF FRACTURE; TRIPLE-PLATE BUTT JOINTS FSCC

followed an irregular course through the third and fourth rows of rivets. About 75% of the fracture was square, with coarse, granular texture, and the remainder was cup-cone or angular-shear with silky or fine granular texture. Near the ends of both specimens, wherever rivets had failed prematurely, the slip was more than 0.25 in.

Triple-Plate Half-Shingle Joints FSCF.—Strains were measured at a limited number of locations along the edges of certain of the plates as shown in Fig. 16(*b*), Diagram *A*. In order that the loads carried by the several plates may be more easily visualized, the curves of Diagrams *C* have been made the same for the near face as for the far face, under the assumption that corresponding plates deformed equally. The assumption was somewhat in error inasmuch as the strain measurements revealed some straightening of initially warped plates.

Computations based on the observed strains indicated that at the given loads a large share of the total load was taken by the rivets in the several rows at the ends of the joints, almost to the beginning of the outer (short) splice plate. At the right end the outer main plates (Plates 3′ and 5′) carried their share of the load practically undiminished from Row 21 to Row 14, whereas at the left end the load in the outer main plates (Plates 3 and 5) gradually diminished. The progress of rivet scaling was in general agreement with the computed distribution of load among rivet rows. Scaling occurred earlier and was much more extensive at the right end in Rows 21 to 19 and in Rows 16 and 15 than in other parts of the joints.



(*a*) DOUBLE-PLATE SHINGLE JOINTS *FSCD*

FIG. 16.—PARTITION OF LOAD AMONG PLATES,

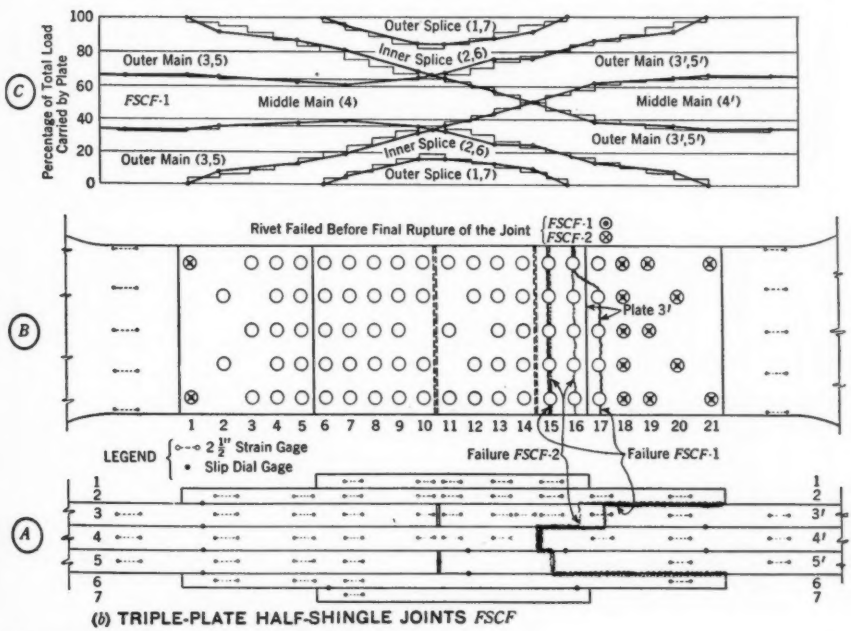
A consideration of all evidence available (strain-gage data, scaling, slip before and after failure, and premature rivet failures) would indicate that near the butts and near the ends of splices (abrupt changes in longitudinal section) the rivets carried relatively high loads.

The joints failed partly in the plates and partly in the rivets, at the end nearest to the butt of the middle main plate (Fig. 16(*b*), Diagrams *A* and *B*). As the rivets in the end rows failed progressively, more load was thrown on the rivets remaining between the end and the butt, and failure resulted. It was observed that, before final failure, rivets sheared up through the fourth row in both specimens. The greater part of the plate fracture was square, with coarse, granular texture, but in Specimen FSCF-1 about 5% of the fracture was cup-cone with silky texture. Wherever rivets failed in the left end, the slip was more than 0.31 in. for Specimen FSCF-1, and more than 0.37 in. for Specimen FSCF-2.

Comparison of Specimens

Efficiency.—The efficiency of the joints at ultimate load ranged from about 71% to 75%, as shown in Table 17, Column (21). The larger joints exhibited the lower efficiencies. There was little difference in efficiency as between butt and shingle joints of the same nominal size. The efficiencies at yield load, shown in Column (15), are 10% to 12% higher than those at ultimate load, but show the same general trend as between joints.

Elongation.—The unit elongation of the single-plate joints was about the same as that of the main plate outside of the joint (Table 17, Column (12)). The



(*b*) TRIPLE-PLATE HALF-SHINGLE JOINTS *FSCF*

AND LOCATION OF FRACTURE; SHINGLE JOINTS

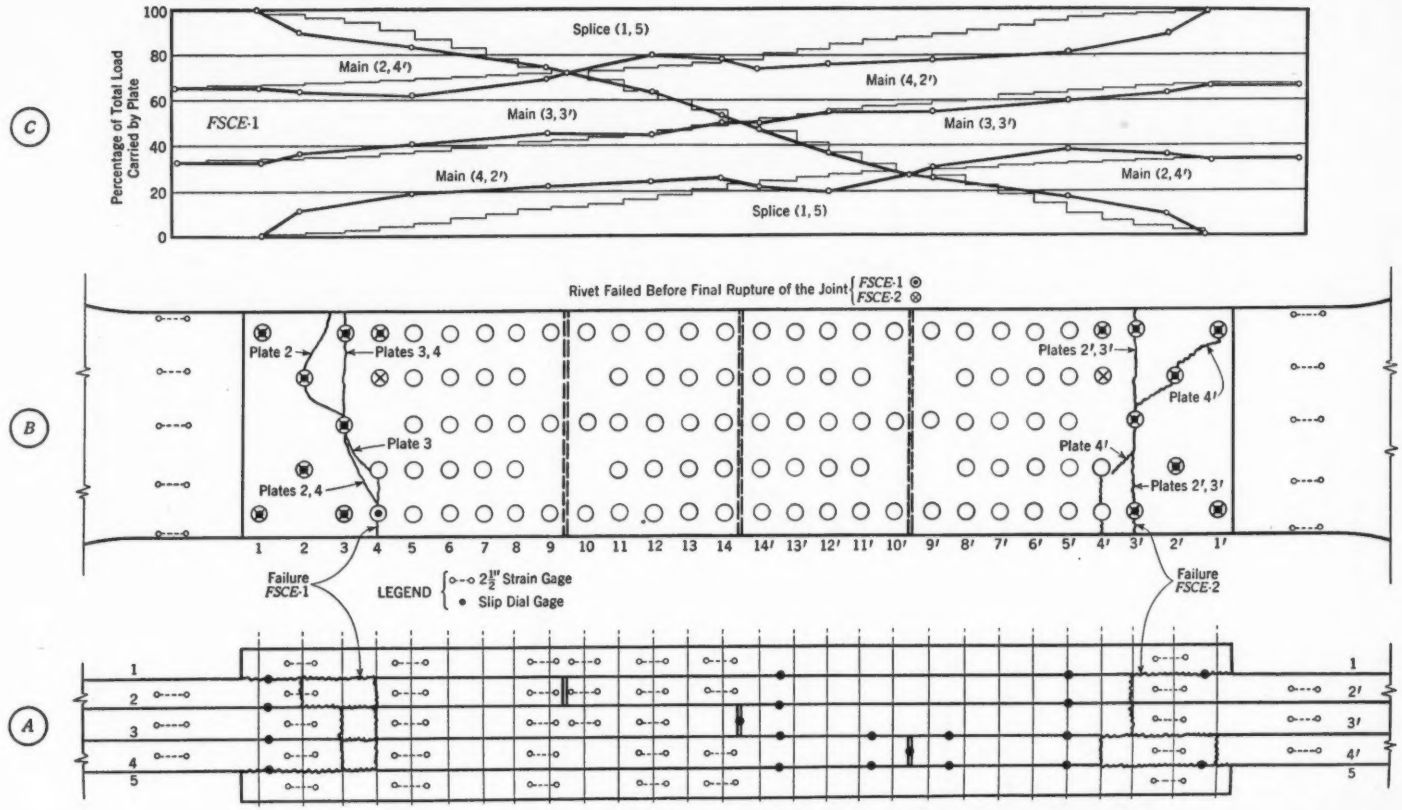


FIG. 17.—PARTITION OF LOAD AMONG PLATES, AND LOCATION OF FRACTURE; TRIPLE-PLATE SHINGLE JOINTS FSCE

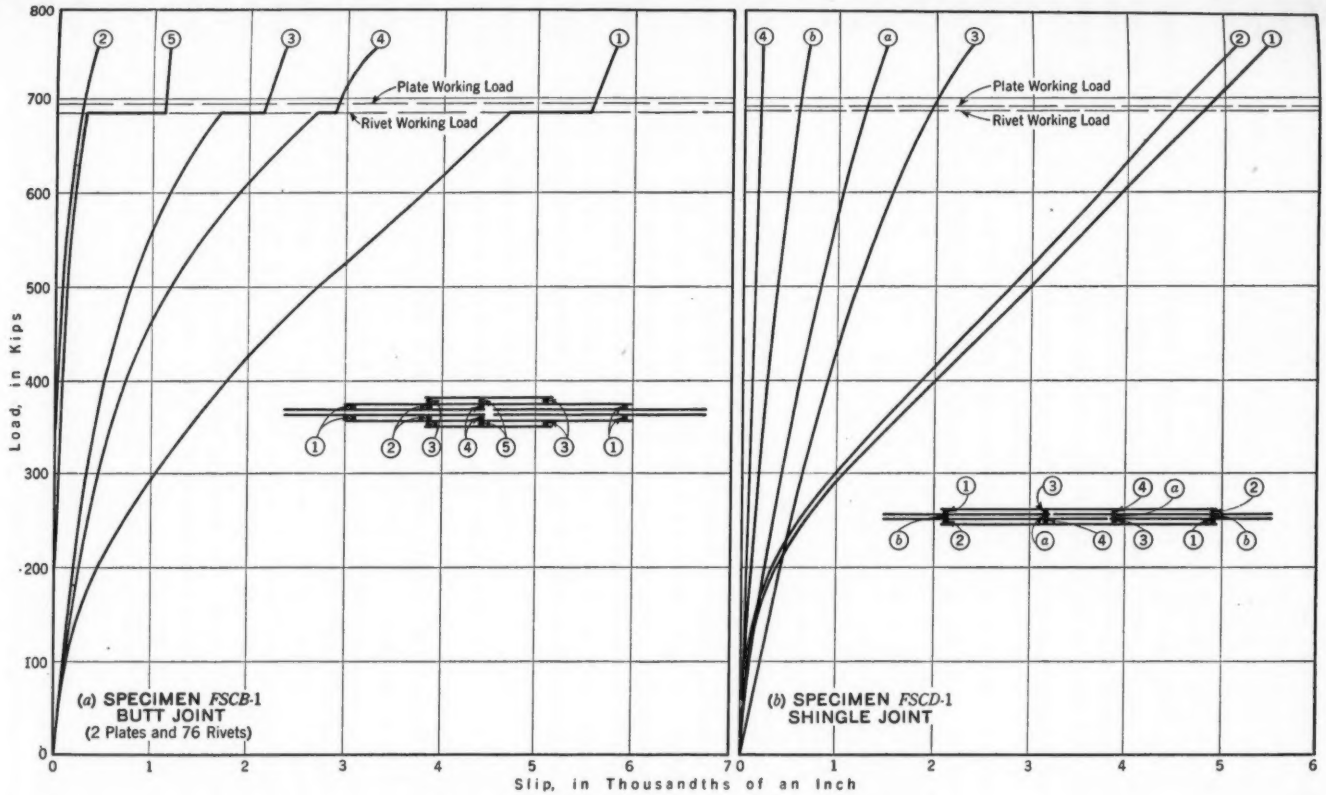


FIG. 18.—LOAD-SLIP RELATIONS; DOUBLE-PLATE JOINTS OF SERIES F

unit elongation of the double-plate and triple-plate joints on the average was 13% less than that of the plate steel. In calculating the elongation of members containing large joints of the types considered herein, for practical purposes the modulus of elasticity of the joint may be considered to be the same as that of the member outside the splice.

Slip.—The end slip at the rivet working load is shown in Table 17, Column (11). It was observed that, on the average, the end slip for the butt joints was about three-fourths of that for the most nearly comparable lap joints (specimens of Series A having silicon-steel plates and carbon-steel rivets). Table 18 shows

TABLE 18.—RELATION BETWEEN AMOUNT OF SPLICE MATERIAL AND END SLIP; SERIES F

Description	Specimen FSCA*	Specimen FSCB*	Specimen FSCD†	Specimen FSCC*	Specimen FSCE†	Specimen FSCF‡
Number of main plates	1	2	2	3	3	3
End slip at rivet working load, in ten-thousandths of inches	71	47	46	76	59	54
Weight of splice material per main plate, in pounds per linear foot of joint	60	40	43	39	31	30

* Butt joint. † Shingle joint. ‡ Half-shingle joint.

that, in general, the greater the amount of splice material per main plate, the greater will be the end slip between main and splice plates. These results were to be expected since, for comparable conditions, in joints having the greater percentages of splice material the splice plates were relatively more rigid.

The load-slip relations for one of each of the double-plate joints are shown in Fig. 18. Inasmuch as the joints of Series F were practically free from the eccentricity that exists in lap joints, and inasmuch as the joints in this series were over-riveted in order to insure plate failures, it was expected that friction between plates would be more of a factor than for comparable lap joints. The difference in friction is evident from the forms of the slip curves for the joints of Series F (Fig. 18) as compared with those for Series A (Figs. 8 to 10 inclusive).

On the whole, short sudden slips, accompanied by snaps, seemed to be more characteristic of the joints in Series F than of those in other series. This behavior is attributed to frictional conditions due to the tendency for plates in a butt joint to remain more closely held together under load than is the case in a lap joint where the plates tend to separate because of bending. In a number of instances in the heavier joints and at the higher loads, sudden slip occurred throughout the length of a joint. Near the ends of the largest joints the slips were relatively large, although they had little effect on the elongation of the joint as a whole. In terms of an "effective" modulus of elasticity of the joint, these movements corresponded at the most to a change from, say, 32 to 29 million lb per sq in.

Transverse Component of Shear in End Rivets.—In this series of joints the two rivets in each end row were placed near the edges of the plate, at a gage distance of 16 in. Owing to lateral contraction of the highly stressed main plates relative to the splice plates (which, at the ends, were subjected to low stress), considerable transverse shearing deformations were induced in these rivets. Although these rivets failed at relatively low computed stresses based

on longitudinal shear, the strength was reasonably close to normal when the resultant of the longitudinal and transverse shear is considered.

Shear in Rivets at Ultimate Load.—In Table 17, Columns (22) and (23), is shown the average shearing stress in the rivets at ultimate load, based both on the total number of rivets and on the number which did not fail prematurely. Although most of the joints failed primarily in the plates rather than in the rivets, it is believed that the rivets were near failure at the ultimate load; in fact, one specimen (FSCC-2) failed entirely in the rivets. The values, in comparison with those of Series A (Table 9, Column (27)) indicate that the resistance per rivet shearing surface was materially less in double shear than in single shear. However, the comparison is not direct, particularly since in Series F the plates were stressed nearly or quite to their ultimate load and, therefore, had elongated greatly and had subjected the rivets near the ends to relatively large detrusions as compared with Series A. Because failure of the rivets was partly due to detrusion, and not entirely to the direct effect of loading, the rivet shearing values in Table 17 should not be used as a general basis for establishing working stresses in double shear.

Conclusions from the Tests

Since the specimens of this investigation were designed to conform reasonably well to standards of practice, the various problems and variables are, of necessity, interwoven. For example, in Series A and F the variations in nominal length of joint involve differences in actual length, thickness (number of plates), grip of rivets, and number of rivets; they are essentially variations in size rather than length. In no case is the width of joint a variable. The conclusions in this section apply only to the conditions and range of these tests.

Herein the term "rivet strength" denotes the average shearing resistance of the rivets in a given group or joint. The strength is computed by dividing the ultimate load by the total cross-sectional area of the rivets in shear, based on the nominal diameter of rivet.

Effect of Length of Joint.—In general, at the ultimate load the greater the length of the joints the lower will be the rivet strength, regardless of the method by which the length was varied (that is, difference in pitch, rivet pattern, number of rivet rows, or over-all size). However, the differences were not large, and minor exceptions arose when the influence of other factors predominated. The general effect of length of connection is summarized in Table 19. For example, for joints with carbon-steel rivets and plates, the rivet strength was only about 5 kips per sq in., or about 10%, less in joints 7 ft long (eighteen rows of rivets) than in joints $2\frac{1}{3}$ ft long (six rows of rivets); for joints with carbon-steel rivets and silicon-steel plates the difference was even less. Furthermore, it appears that small variations in length are not important in comparison with variations arising from fabrication inequalities.

As between the butt joints of different length (FSCA, FSCB, and FSCC), the longer joints not only were lower in efficiency but also they failed essentially in the rivets rather than in the plates (Table 17).

Effect of Rivet Pitch.—From the tests of Series B, in which the pitch between transverse rivet rows was the only variable, it appears that over the range

investigated (3 to 6 in.) the effect of pitch was small, although the 4.5-in. pitch (4.5 rivet diameters) gave slightly higher efficiency than either the 3-in. or the 6-in. pitch.

TABLE 19.—SHEARING STRENGTH OF RIVETS

Plate steel	Carbon-Steel Rivets							Manganese-Steel Rivets						
	Specimen number*	Number of:		Grip, in inches	Shear Stress in Rivets, in Kips per Square Inch			Specimen number*	Number of:		Grip, in inches	Shear Stress in Rivets, in Kips per Square Inch		
		Rivets	Rivet rows		At effective rivet yield	At yield load	At ultimate load		Rivets	Rivet rows		At effective rivet yield	At yield load	At ultimate load
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Carbon	ACC2†	2	2	1¼	38.0	....	60.8	ACM2†	2	2	1¼	....	....	79.8
	GCC4†	4	2	1¼	38.0	....	62.1	GCM3†	3	2	1¼	....	....	93.6
	CCC7	8	2	⅝	....	44.6	57.0	....	....	....	....	....	....	....
	DCC7	14	2	1¼	....	37.7	56.6	....	....	....	....	....	....	....
	BCC20*a*	22	6	1½	....	39.6	56.5	....	....	....	....	....	....	....
	BCC20*b*	22	6	1½	....	39.8	58.6	....	....	....	....	....	....	....
	BCC20*c*	22	6	1½	....	37.6	57.2	....	....	....	....	....	....	....
	ACC18	18	6	1½	31.8	38.9	55.7	ACM12	12	4	1½	45.4	59.0	75.9
	ACC36‡	36	12	1½	32.3	36.4	55.1	ACM24‡	24	8	1½	49.3	56.8	77.4
	ACC54§	54	18	2½	35.4	37.7	50.0	ACM36§	36	12	2½	47.8	56.5	71.4
Silicon	ASC2†	2	2	1¼	35.0	....	57.2	ASM2†	2	2	1¼	....	....	72.9
	GSC6†	6	2	1¼	35.0	....	55.7	GSM4†	4	2	1½	....	....	70.3
	ASC18	18	6	1½	....	40.0	54.8	ASM12	12	4	1½	45.3	59.8	72.4
	ASC36‡	36	12	1½	32.3	41.7	54.0	ASM24‡	24	8	1½	47.5	63.3	78.0
	ASC54§	54	18	2½	32.2	38.9	51.1	ASM36§	36	12	2½	45.6	57.3	72.7
Nickel	....	....	....	....	....	....	....	ANM2†	2	2	1¼	....	....	86.0
	....	....	....	....	....	....	....	ANM12	12	4	1½	41.9	64.2	74.7
	....	....	....	....	....	....	....	ANM24‡	24	8	1½	46.7	67.9	77.7
	....	....	....	....	....	....	....	ANM36§	36	12	2½	49.3	65.8	75.7

* One-plate specimens, with exceptions noted. † Pilot tests at the University of Illinois. ‡ Two-plate specimen. § Three-plate specimen.

Effect of Kind of Rivet Steel.—As compared with strength of rivets in small joints (pilot tests) or in a direct shear test of undriven rivets, in the large joints of Series A the manganese-steel rivets performed somewhat better as regards ultimate strength than the carbon-steel rivets. Although this difference is attributed in part to the difference in actual length of joint for a given nominal length, it seems fair to credit the manganese-steel rivets with this advantage since fewer rivets are needed. For the joints of Series A, all of which failed in the rivets, the average ratio of observed strength to the assigned rivet working stress (that is, the "factor of safety") was found to be 3.73 for the manganese-steel rivets as compared with 3.52 for the carbon-steel rivets.

Effect of Kind of Plate Steel.—In the joints that failed in the rivets, the effect of kind of plate steel was not large or consistent, although the decrease in shearing strength of rivets with length was somewhat more marked in the case of the carbon plate steel than with the less ductile plate steels. There was less tendency for premature failure of end rivets with the less ductile plate steels.

Single Shear vs. Double Shear.—Although the comparison is not direct, being based on the tests of joints of Series A and F [see heading: "Tests of

Butt and Shingle Joints; Series F (Comparison of Specimens): Shear in Rivets at Ultimate Load] the indications are that the strength of a rivet surface in double shear may be as low as 80% of the strength in single shear. These indications agree with the findings in the pilot tests[8] where non-driven rivets were tested in both single and double shear.

Net Section.—An outstanding finding of these tests is that the strength of the plates in a riveted joint is not directly proportional to the area of the net section, as commonly assumed in practice. The use of fewer rivets in the end rows than in interior rows is ineffective, as shown by the tests in Series C and D. Wide spacing of the end-row rivets (near the edges of the plate) is especially undesirable; in the tests of Series F it was found that differential lateral contraction of the opposing plates at each end of the joint subjected such rivets to lateral strain which, combined with the longitudinal strain, caused premature failures of these rivets.

In tests of joints having the full number of rivets in the end row (Series C and D), the efficiency was about 10% higher when the gage was about four rivet diameters than when a gage of about three rivet diameters was used.

The experimental results bore no consistent relation to the specification requirements for determining net section. In these tests, there was little difference in strength as between joints having an end pitch of four to six rivet diameters and joints having an end pitch of two to three rivet diameters, although the specifications would assign a smaller computed net section to the latter group. Regardless of the number of rivets in the end row, the net section found by deducting the projected sectional areas of rivet holes in the first two rows correlated better with the observed strengths than did the net section computed by standard specifications. It is apparent that there is nothing to be gained by using an elaborate formula to evaluate the effect of pitch on net section for joints of this type.

Butt vs. Shingle Splices.—There was little difference in efficiency as between the butt and shingle splices of Series F. The triple-plate shingle splices require less material for fabrication than the butt splice, as shown in Table 20;

TABLE 20.—MATERIAL REQUIRED FOR VARIOUS TYPES OF SPLICE; SERIES F

Description	Butt splice FSCC	Shingle splice FSCE	Half-shingle splice FSCF
Weight of splice plates, in pounds	850	684	496
Number of rivets	114	114	85

this advantage, however, may be offset by the somewhat greater difficulty of field erection for shingle splices. The partition of load between the splice plates agrees fairly well with the usual assumptions in designing these splices.

Slip.—The relative movement, or slip, of adjacent plates is of interest in connection with the behavior of riveted joints. In fact, the rivets cannot develop resistance to shear until after slip occurs. Slip is believed to be of special importance with regard to structural members subjected to repetitions or reversals of load.

The load-slip relationship may be divided into these four stages: (I) Load is transmitted by friction between the plates, and general slip throughout the joint does not occur; (II) slip begins and the rivets are coming progressively into bearing; (III) the rivets are in full bearing and the load-slip relation is linear; and (IV) plastic action and yield occur in rivets, plates, or both (see heading "The Action of Riveted Joints").

These stages of joint action are not all developed to the same extent during the loading of joints of various types. In joints having four to six rows of rivets, the end of Stage I may occur at a rivet stress of 10 kips per sq in. or less. For the single-plate lap joints of Series A, the ratios of the loads at the beginning of Stage III (as nearly as could be determined) to the rivet working loads are shown in Table 13, Columns (6) and (11).

The ratio of slip measured near the ends of the joints to slip measured at the middle of length of a lap became a constant when the rivets came into full bearing (Stage III). The ratio was a function of the length of the joints.

In the tests of Series A, the joints containing carbon-steel rivets exhibited markedly less slip than those containing two-thirds as many manganese-steel rivets. As between the various series, the slip was less for joints with full-row riveting than for joints with any other rivet pattern.

Partition of Load Among Rivet Rows.—Under the assumption of perfectly elastic behavior of an ideal joint, in joints containing more than two rows of rivets, the rivets in the end rows are subjected to more than the average load per rivet.[9] In these tests, the results of strain measurements made over the working range showed that in actual joints the same tendency exists but that inequalities arising from fabrication conditions cause irregular and erratic load partition, and that the irregularities may be large as compared with the differences in load partition called for by the elastic theory.

The following general conclusions are drawn from various phases of the tests bearing on the distribution of stress in riveted joints:

(1) At low loads (Stage I), the load partition among rivets may be controlled largely by such conditions within a joint as warping of plates, variations in initial tension of rivets, and character of the faying surfaces;

(2) After slip becomes general throughout a joint (Stage II), there is a tendency for the shearing stresses to become equalized among the rivets in the various rows, although uniform partition of load is not probable;

(3) From limited data available, it appears that in Stage III the rivets in the end rows tend to take more than the average load per rivet;

(4) Although no direct evidence is available, it seems likely that in the early part of the yield range (Stage IV), the load partition among the rivets tends to become uniform; and,

(5) In the latter part of the yield range, the amount of plastic flow in the plates is so great that the resulting elongation is sufficient to cause detrusion of end rivets. For $\frac{7}{8}$-in. rivets, this rivet failure occurs when the end slip is about 0.3 of the rivet diameter, and occurs more frequently in the longer joints and in those made with the more ductile steels. It does not necessarily indicate that these end rivets are taking more than their proportionate share of the load.

In general, although it is probable that the partition of stress among rivets is not uniform at any stage of loading, the test results indicate that an assumption of uniform partition is as reasonable as any that has been made.

Recommendations Applicable to Design

The results of these tests, together with those of other investigators, appear to warrant the following statements relative to practice in the design of riveted joints for bridge construction:

(1) Nothing is gained by an attempt to detail a tension member with a critical net area greater than about 75% of the gross area;

(2) There is no justification for elaborate formulas to calculate the effect of rivet stagger on net section;

(3) Joints should be as compact as practicable (optimum results will probably be obtained by full-row riveting with a gage of about 4.5 rivet diameters and a pitch of 3.5 to 4 rivet diameters);

(4) Statements (1) to (3) lead to the suggestion that the allowable loads on riveted tension members be based on, and expressed in terms of, stress in the gross section; and, that the net section be not less than 75% of the gross section;

(5) The practice of assuming equal shear per rivet, regardless of length of joint, is satisfactory;

(6) Except in comparatively heavy structures where reduction in size or weight of splice is important, there is little reason for using manganese-steel rivets rather than carbon-steel rivets. Owing to the greater slip that occurs in joints with manganese-steel rivets, carbon-steel rivets are preferable in members subject to stress reversals; and,

(7) Working stresses recommended for use in the design of riveted joints in bridges, for steels commonly specified, are shown in Table 21. These

TABLE 21.—Recommended Working Stresses for Riveted Bridge Joints

Property	Steel	Minimum Strength in Tension of Mill Coupons, in Kips per Square Inch		Recommended working stress, in kips per square inch	Ratio
		Yield	Ultimate		
Tension in gross section of plate	Carbon	33.0	60.0	16.0	1.00
	Silicon	45.0	80.0	20.0	1.25
	Nickel	55.0	90.0	22.5	1.40
Single shear in rivets	Carbon	28.0	52.0	15.0	1.00
	Manganese	42.0	75.0	20.0	1.33
Double shear in rivets	Carbon	28.0	52.0	13.5	0.90
	Manganese	42.0	75.0	18.0	1.20

recommended stresses were developed from a consideration of: (a) The strengths of the steels as indicated by standard mill acceptance tests; and (b) the relations between the strengths of mill coupons and the strengths of joints of structural proportions. The relationships between the strengths, in tension, of mill specimens and the properties of structural joints, based on a study of

data from various sources, are as follows:

Ratio	Tension in gross section of plate	Single shear in rivets (nominal diameter)
$\frac{\text{Useful limit point for a joint}}{\text{Yield strength of mill coupons}}$	0.72	....
$\frac{\text{Useful limit point for a joint}}{\text{Ultimate strength of mill coupons}}$	0.42	....
$\frac{\text{Ultimate strength for a joint}}{\text{Ultimate strength of mill coupons}}$	0.75	0.88
$\frac{\text{Effective rivet yield for a joint}}{\text{Yield strength of mill coupons}}$	....	0.83
$\frac{\text{Effective rivet yield for a joint}}{\text{Ultimate strength of mill coupons}}$	....	0.55

In computing these values, the useful limit point of plates in a riveted joint was assumed to be 85% of the yield strength of the joint.

The foregoing ratios are applied to the strengths of mill coupons to obtain the effective strengths of the materials acting in structural joints, as given in Table 22. The selection of strength values for mill coupons is based on an examination of mill reports on tests of steels of the given grades. It is believed that the foregoing ratios are conservative. Inasmuch as—to avoid rejections—the mills endeavor to supply material somewhat above the minimum specified, it should be safe to take values midway between the minimum and the average. Values taken in this manner are shown in Table 22 as "expected strengths."

TABLE 22.—EFFECTIVE STRENGTHS OF MATERIALS ACTING IN STRUCTURAL JOINTS

Steel	Expected Strength in Tension of Mill Coupons, in Kips per Square Inch		Useful Limit Point (for Plates) or Effective Rivet Yield (for Rivets)				Ultimate Strength	
	Yield	Ultimate	Based on yield, kips per square inch	Based on ultimate, kips per square inch	Average, kips per square inch	Ratio	Kips per square inch	Ratio
(*a*) Tension in gross section of plate								
Carbon	36.3	63.3	26.1	26.6	26.3	1.00	47.5	1.00
Silicon	47.6	83.7	34.3	35.2	34.7	1.32	62.9	1.32
Nickel	59.6	94.9	42.9	39.8	41.3	1.57	71.0	1.50
(*b*) Single shear in rivets (nominal diameter)								
Carbon	32.6	54.7	27.0	30.0	28.5	1.00	48.1	1.00
Manganese	49.8	81.4	41.4	44.7	43.0	1.51	71.7	1.49

To obtain the recommended working stresses for carbon plate and rivet steels (Table 21), the strengths in Table 22 have been reduced by using a factor of about $1\frac{3}{4}$ in the case of the useful limit points and about 3 in the case of the ultimate strengths. When high-strength plate steels are used in place of

ordinary carbon steel, some desirable property must be encroached upon to secure the added strength. The ductility and plastic flow of these high-strength steels are less than those of carbon steel, so that there is less opportunity for stress readjustment to take place and secondary stresses become relatively more important. For these reasons, working stresses should not be increased in proportion to strength. Likewise, the values of working stresses for manganese-steel rivets as compared with those for carbon-steel rivets should be lower than their relative strengths because of the greater slip that occurs in joints containing manganese-steel rivets. The rivet values in double shear per shearing surface have been taken as 0.90 times those for single shear.

Acknowledgments

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The specimens were fabricated under contract at the Ambridge plant of the American Bridge Company from steel manufactured at the Homestead plant of the Carnegie-Illinois Steel Corporation. Of the coupons taken at the time the specimens were manufactured, about one-third were tested by the Carnegie-Illinois Steel Corporation under the supervision of J. M. Bailey, Principal Steel Inspector for the San Francisco-Oakland Bay Bridge; the remaining coupons were tested at the University of California.

The static tests on large joints were made in the Engineering Materials Laboratory of the University of California, during the period 1935 to 1937. The University of California was represented by Charles Derleth, Jr., M. Am. Soc. C. E., Dean of the College of Engineering, and Raymond E. Davis, M. Am. Soc. C. E., Professor of Civil Engineering, in charge of the Laboratory. The following members of the staff of the Engineering Materials Laboratory, University of Illinois, participated in the conduct of the tests and the analysis of the results: J. L. Banville, E. H. Brown, Harmer E. Davis, Assoc. M. Am. Soc. C. E., H. D. Eberhart, Jun. Am. Soc. C. E., E. J. Garbarini, Jun. Am. Soc. C. E., Norman Haavik, J. W. Kelly, M. Am. Soc. C. E., Alexander Klein, Assoc. M. Am. Soc. C. E., J. F. Ryan, G. E. Troxell, M. Am. Soc. C. E., and E. L. Whittier. During the progress of the work, the San Francisco-Oakland Bay Bridge was represented by C. C. Winter, Assoc. M. Am. Soc. C. E., E. B. Payne, and R. L. Richardson, Juniors, Am. Soc. C. E.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STREAM POLLUTION IN THE OHIO RIVER BASIN A SYMPOSIUM

Discussion

BY MESSRS. W. L. STEVENSON, AND D. E. DAVIS

W. L. STEVENSON,[49] M. AM. SOC. C. E. (by letter).[49a]—The discussions and subsequent experiences during the 1937 flood in the Ohio River, strengthens the writer's conviction that preparedness is essential for protection of the public health during floods. This means pre-planned general principles of procedure, trained personnel in Federal, State, and local health departments, systematic swift mobilization of skilled men and women with essential equipment and materials, and resourceful and trained water-works operators. Experience has also shown the need for more extensive and better facilities to those charged with the duty of forecasting flood crests so that due warning may be given and safe-guarding measures adopted where possible. Of equal importance is the need for strengthening the confidence of the public in health authorities in order that there may be the fullest co-operation.

In the final analysis, it is upon the large number of lay people whose homes are in the flooded areas that the burden falls. The most that the small group of trained engineers, physicians, and nurses can do in such emergencies is to direct, guide, advise, and help.

Mr. Holmquist, writing from personal flood experiences in New York State "emphasizes in the strongest terms" the responsibility of the local authorities; and Mr. Weston refers to the serious condition that arises when local authorities lack comprehension. Many a ship reaches its port safely in fair weather, but when storms arise and the winds rage, only a trained captain and crew can save the ship. So it is in emergency sanitation during flood or disaster.

NOTE.—The Symposium on Stream Pollution in the Ohio River Basin was presented at the meeting of the Sanitary Engineering Division at Pittsburgh, Pa., on October 14, 1936, and published in January, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1938, by Messrs. D. E. Davis, L. S. Morgan, C. A. Holmquist, and Robert Spurr Weston; May, 1938, by Messrs. Karl Imhoff, Henry D. Johnson, Jr., Charles M. Reppert, Edwin K. Morse, James H. Le Van, and John C. H. Lee; and June, 1938, by Messrs. W. H. Wisely, F. C. Dugan, G. M. Ridenour, Arthur W. Baum, Jr., G. R. Scott, and Don E. Bloodgood.

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[49a] Received by the Secretary November 21, 1938.

The unqualified "boil water" notices used by the Pennsylvania Department of Health during the March, 1936, floods were criticized by some water purveyors, and Mr. Holmquist discussed this question constructively. The writer maintains his position in this matter, believing that the obvious injustice to those water purveyors who remain able to serve pure water during the emergency is far outweighed by the extra safeguard to the public health of the unqualified warning to boil all water used for drinking within the flooded areas.

It was common experience to find some private water wells within flooded areas which were dominantly furnished by a piped public water supply that remained uncontaminated during the flood. Therefore, it is better for the many to boil the pure piped water supply unnecessarily than for the few to drink, unboiled, the contaminated water from the flooded private wells.

As the flood waters receded in Pennsylvania after the March, 1936, floods, one of the heaviest tasks of the State Health Department was to direct the cleansing of polluted private wells. There was nothing spectacular about this work, such as the heroic service of many employees of public water-works menaced by the flood as the rivers crested; but it was a vital part of the entire program for prevention of water-borne disease.

D. E. DAVIS,[50] M. AM. SOC. C. E. (by letter).[50a]—Certain threads of thought run through many of the papers presented in the Symposium, and in the comments by the various discussers, that indicate a fairly general agreement on certain fundamental concepts. It is recognized that a broad drainage area is the proper geographical unit for approaching the solution of pollution problems, and that in many instances the co-operation of several States may need to be enlisted to make this possible. State compacts appear to offer favorable mediums for action.

A unified approach also offers advantages in the cleaning up of pollution in certain metropolitan districts, such as Pittsburgh, Pa., and Cincinnati, Ohio, where a number of separately organized communities are affected by similar conditions. Sewerage Boards, serving the group of communities and with power to raise money, assess costs, and collect rentals, offer a hopeful approach.

The rôle of research in defining the extent of pollution and the degree of purification required is highly essential. This tool can also be utilized in solving problems connected with self-purification in streams, as well as those of a special character having to do with the underlying phenomena of the action of acid on sewage.

Grants-in-aid by the Federal Government have resulted in a marked increase in the total number of treatment plants during the past few years, and the continuation of this policy in some form offers hope of further advances. All are agreed that the public in general is taking a keener interest in the vital job of cleaning up its streams, and that with a little encouragement a large measure of accomplishment may be expected.

[50] Engr. (The Chester Engrs.), Pittsburgh, Pa.

[50a] Received by the Secretary February 14, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN OF PILE FOUNDATIONS

Discussion

BY C. P. VETTER, M. AM. SOC. C. E.

C. P. VETTER,[36] M. AM. SOC. C. E. (by letter).[36a]—The generous contributions which, in the form of discussions, have been made to the art and practice of the design of pile foundations, have been most gratifying. Much new light has been thrown on the various principles involved, and the writer wishes to express his appreciation.

Mr. Hill develops formulas for pile loads for the case in which all piles are parallel with each other. Apparently, it has been overlooked that under the assumption of the exterior load being resisted solely by axial pile reactions, such a foundation will be stable only if the resultant of the exterior loads is parallel with the piles. This may be seen directly from Equations (46*a*) and (46*b*) which by division yield:

$$\frac{H}{V} = \tan \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (91)$$

Such a pile foundation is only rarely encountered in practice, except in the case of vertical piles supporting a vertical load ($\alpha = 0$).

Mr. Hill extends the investigations to foundations having piles in opposite directions with equal (but opposite) batters. Equation (52) is believed to cover the case correctly. However, it is incorrect to apply this formula to the writer's numerical example, in that Mr. Hill seems to have neglected the vertical piles in the example. Presumably, this accounts for the deviation in numerical values of the pile loads given in Mr. Hill's tabulation following Equation (52).

Mr. Hill next discusses pile foundations for which the resultant of the exterior forces does not fall in the plane of symmetry of the pile group and is not even parallel with that plane. The writer wishes to warn prospective designers against taking too much for granted when considering pile founda-

NOTE.—The paper by C. P. Vetter, M. Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Hibbert M. Hill, and Odd Albert; June, 1938, by Messrs. August E. Niederhoff, A. A. Eremin, and Jacob Feld; September, 1938, by John M. Coan, Jr., Jun. Am. Soc. C. E.; January, 1939, by A. Agatz, Esq.; and February, 1939, by D. P. Krynine, M. Am. Soc. C. E.

[36] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[36a] Received by the Secretary March 27, 1939.

tions and loadings arbitrarily distributed in space. The problem is "tricky" and highly complex. It has been solved to perfection by Dr. Nökkentved[5] whose work should be consulted before important designs in this line are undertaken.

Mr. Hill further states that it is desirable that a foundation utilizing batter piles contain piles battered in opposition to each other. The writer agrees fully with this assertion. In fact, in the design of the pile foundations for Imperial Dam on the Colorado River (perhaps the most extensive pile-driving undertaking in the United States containing battered piles), all foundations had piles in opposite directions. Fig. 1(*a*) shows a typical bent of a part of Imperial Dam.

Professor Albert draws attention to the valuable contributions made by Swedish engineers, about thirty or more years ago, to current knowledge of the behavior of pile foundations. The writer gave proper reference in his paper to those early pioneers in the field. Mr. Albert prefers Hultin's graphical method for the determination of pile loads for the case of hinged piles, and the writer agrees that, for one thoroughly familiar with that method, results may possibly be obtained in less time by the use of that method than by the formulas given in the paper. The difficulty with Hultin's method, as with nearly all the older methods, is that, although the applications may be speedy after the method has been mastered, the method itself is so intricate that considerable time must be spent by the designer to familiarize himself with the subject or to "brush up" on it even if he has known it once. To most engineers the designs of pile foundations come only rarely, and it would seem of considerable importance that the principles used in such designs should be formulated so simply that actual numerical work may be undertaken with the least possible delay. The expert who does little besides designing pile foundations would do well, and possibly would save time, by studying some of the other works on the subject, as referred to in the paper. He should ascertain, however, that the method he proposes to utilize rests on a sound theoretical base. The references given by the writer and by Mr. Albert are believed to be reliable in this respect.

Mr. Niederhoff appears to be mostly concerned with defending the still widely employed practice of designing pile foundations which depend for the stability of the structure on lateral resistance of the upper layers of the foundation material. The writer is definitely of the opinion, and so stated in his paper, that if the upper layers are of such consistency that they may safely be relied upon for lateral resistance, piles are not needed and should not be used. The reason is that, in driving the piles, the original firmness of the upper layers of the soil is often destroyed and so is the soil's capacity for resisting lateral loads.

Mr. Niederhoff would seem, in places, to have misunderstood some of the points brought forth in the paper, possibly because of lack of clarity on the part of the writer. The statement is made that "His [the writer's] analysis also appears needlessly complicated and leads to values that are different from

[5] "Beregning af Pälevärker," by C. Nökkentved, Copenhagen, 1924.

those obtained by the writer [Mr. Niederhoff] using only the resolution of forces into components that act upon the piles axial and normal to the axis." The writer maintains that an analysis is "needlessly complicated" only when the same result, or a result equally correct, may be obtained by some other analysis which is easier, less complicated, or more readily understood. That results, different from the writer's, are obtained by Mr. Niederhoff's analysis is not surprising, in that he satisfies the equilibrium conditions only, without paying any attention to deformations or displacements. The relationship between a pile's axial compression and its lateral displacement does not depend only on the pile's batter and the direction of the resultant. It depends most of all on the location of the resultant with respect to the top of the piles. The movement of the resultant laterally from one side of the elastic center to the other may reverse the direction of the lateral displacement of the pile heads and, as a result, reverse the direction of the lateral resistance. It was one of the main purposes of the paper to demonstrate this, possibly surprising, but nevertheless unquestionable, fact.

Mr. Niederhoff further objects to the writer's statement that "In many practical cases, such as bridge piers and gate piers in hydraulic structures, the error introduced by this assumption [that of hinged ends] is insignificant." The sentence that follows immediately in the paper may have been overlooked: "However, in some applications, such as pile bents for trestles, the error becomes of considerable magnitude."

Under "Pile Foundations Which Are Symmetrical About a Vertical Line: Comment," the writer stated:

> "* * * if, therefore, the piles, approximately, go through the same point the rotation for a given moment would be large. The assumption that the effect of restraint may safely be neglected does not hold for this case and the stability of the pier depends almost entirely on the restraint and the stiffness of the piles."

A group of vertical test piles, as referred to by Mr. Niederhoff, may be said to represent a pile bent at its worst, in that all piles intersect at the same point (infinity). That less deflection was found for the restrained test piles than for hinged ones would seem to bear out the writer's statement rather than to refute it.

In determining the depth at which the pile is restrained (necessary for the cases where restraint is important) Mr. Niederhoff prefers to follow the analysis by A. E. Cummings, M. Am. Soc. C. E.[23] The writer can see no objection to the use of Mr. Cummings' formulas as a guide. These formulas are based on very special assumptions, probably applicable to a test pile, but only applicable to a very limited extent to piles in an actual foundation. One of the assumptions in question is that the point of contraflexure occurs at half depth. This assumption applies neither to a test pile nor to a pile in an actual foundation. However, it affects the final result very little. Of greater consequence is Mr. Cummings' assumption that the tangent to the pile's deflection curve remains vertical at the top. Any structure supported on piles will rotate under load, unless the resultant of the exterior forces goes through the elastic

[23] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 255.

center. The more nearly the piles intersect at a point, the greater the rotation will be, and the greater will be the error in assuming the tangent at the tops of the piles to be vertical. From a mathematical point of view, the analysis of the same problem by Y. L. Chang[37] is preferable to that of Mr. Cummings. Mr. Chang uses a somewhat different modulus of elasticity of the soil, but that might possibly be remedied.

Mr. Eremin suggests direct algebraic solution of the three simultaneous equilibrium equations. The method is direct but somewhat tedious and time-consuming in the case of many piles. Furthermore, the sign conventions become of primary importance since there is no direct check on the work as it progresses.

Mr. Feld submits a number of general statements regarding the behavior of piles under load. In every case he seems to conclude that the behavior of a pile foundation consisting of many piles is similar to the behavior of a single pile subject to a similar load. Such conclusions are not permissible. The case can best be illustrated by an example. Assume a batter pile, *A B*, acted upon by a horizontal force, *P* (Fig. 24). Mr. Feld uses exactly this case as an illustration. In order for the pile to be in equilibrium, at least two forces, *R* and *S*, are required. They must be produced by the resistance of the soil or, in some equivalent manner, by the stiffness and embedment of the pile. If the soil offers insufficient resistance to lateral displacement, or if the pile lacks stiffness or strength, the structure will fail. If, now, a second pile, *A C*, is provided, the lateral forces *R* and *S* are not required for equilibrium and no reliance need be placed on the lateral resistance of the soil or on the stiffness and flexural strength of the piles. All that is necessary is that the piles can sustain axial loads such as *Q* and *T*. That *T* is tension is of no particular importance; as a rule there are sufficient vertical forces also acting on the structure to counterbalance the tension. It is believed that this simple example presents the advantage of battered piles in the simplest possible manner and that it answers virtually all the questions raised by Mr. Feld.

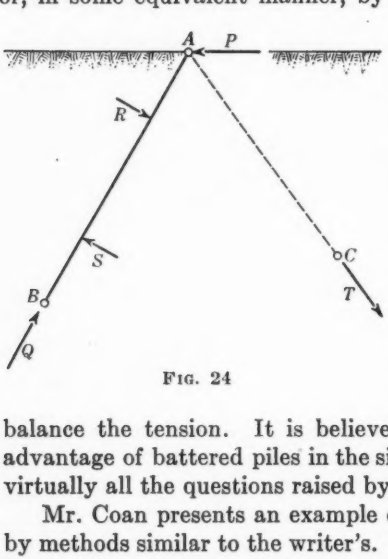

FIG. 24

Mr. Coan presents an example of a three-dimensional pile group analyzed by methods similar to the writer's. The results in this particular case seem to agree fairly well with model test results. However, as mentioned previously, great caution should be exercised in applying the writer's methods to spatial pile groups and loading conditions.

Professor Agatz presents a very ingenious method for direct determination of pressures in pile foundations consisting of piles in three different directions. His method is entirely algebraical, whereas the writer's method, in the general case, is partly algebraical and partly graphical. The theoretical background

[37] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 272.

for both methods is, needless to say, the same. For the special case in which one of the three pile groups is vertical, complete formulas ready for use are presented. The only criticism the writer would venture in connection with this discussion is the use of numerical values instead of algebraical values for the ratio m ($|m|$) and the angles α ($|\alpha|$). As a general rule the writer believes this to be a dangerous practice. Sign errors constitute a surprisingly great proportion of the errors made in engineering calculations and, at times, such errors are very difficult to detect. For that reason the writer is in favor of definite sign conventions and strict adherence to them. As far as Professor Agatz' discussion is concerned, this is, of course, a minor matter. His equations, although complicated in appearance, are not as complicated in actual use as might be expected. In general, the writer prefers methods which present a little easier avenue of approach but, since Professor Agatz' method is primarily applicable to retaining walls and quay walls with considerable duplication, the designer can well afford to spend considerable time in becoming familiar with this special method.

Professor Krynine calls attention to the fact that the writer's method produces pressures on each pile but not settlements on the structure due to these pressures. This deficiency was recognized in the "Synopsis" of the paper, from which is quoted: "It is the subsoil that 'carries' the load, and consolidation of soft material below the piles will inevitably contribute its share to the settlement of the structure whether or not piles are used," and further, "It may be argued * * * in particular, that considerations from the point of view of soil mechanics have been neglected in favor of those of purely structural mechanics. This argument cannot be refuted in a general manner." It would be equally futile to attempt to refute Professor Krynine's argument. The purpose of the paper was restricted and was limited to the determination of pile stresses, and, in general, to the encouragement of the use of battered piles for structures subject to horizontal loads. It did not deal, primarily, with the settlement of structures, and to attempt to predict such settlements solely from the pile stresses would obviously be erroneous. Settlements due to causes other than the elastic deformations of the piles will probably affect the pile stresses, particularly in cases where the restraint of the piles is important. Hence, contrary to the apparent opinion of some of the discussers, the methods proposed by the writer are best suited for pile foundations in which the restraint of the piles is of minor importance; that is, large bridge piers and piers for hydraulic structures. In such structures the pile stresses are likely to be least affected by secondary earth movements.

Only one other of the points raised by Professor Krynine needs to be discussed further; namely, that of the distribution along the pile's axis of the frictional forces. This question was also raised by Mr. Niederhoff. The proposed use of the triangular distribution has a very simple explanation. Assume that the coefficient of friction between the pile and the soil is constant and independent of the normal pressures and of the rate and magnitude of the relative movements. The coefficient of friction between solid substances is generally assumed to have just those characteristics. The frictional resistance to the movement of the pile would thus be directly proportional to the lateral

earth pressure which, again, is usually assumed to be rectilinearly increasing with depth. As a result, the friction per unit area would increase rectilinearly with depth and, if the pile is cylindrical or prismatic, the friction per unit length of pile would also increase rectilinearly with depth. In the case of a tension pile, conditions would be exactly the same except that the normal earth pressure, according to the familiar Coulomb-Poncelet earth-pressure theory, is less in the case of a vertically upward movement of the pile than it is for a vertically downward movement. The writer does not agree with Professor Krynine that a consistent conclusion would be that, in the case of a tension pile, the frictional force would be maximum at the top and zero at the tip of the pile.

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DISCUSSIONS

OBSERVED EFFECTS OF GEOMETRIC DISTORTION IN HYDRAULIC MODELS

Discussion

BY KENNETH D. NICHOLS, JUN. AM. SOC. C. E.

KENNETH D. NICHOLS,[8] JUN. AM. SOC. C. E. (by letter).[8a]—The frank criticism and constructive comments in the discussions are appreciated. The writer is in practically complete agreement with the views expressed by Captain Ehrgott, and his discussion aids materially in clarifying the original paper. He presents some interesting and valuable data on the observed values of n in small models. The observation and gradual collection of more of such data will greatly facilitate the work of the model designer in the future.

Captain Ehrgott implies that Equation (4) might be applicable in the design of movable bed models. In the writer's opinion Equations (3) and (4) have little application in the average movable-bed model study. They are intended primarily for consideration in design and operation of fixed-bed models used for hydraulic studies. In the average movable-bed model of a river the bed-movement considerations are so much more important than certain hydraulic requirements that the latter must be subordinated. Only in certain types of river tidal models do hydraulic and bed-movement requirements necessitate equal consideration. In general the purpose of the model should be kept firmly in mind and the study confined to rather narrow limits. This limitation has brought about the necessity for having different techniques for fixed-bed models and for movable-bed models.

The writer is in complete agreement that Equation (3) is not Froude's law, but an equation of compatibility between Froude's law and Manning's formula. Conclusion (3) of Item No. 6 is a comparison of the empirically determined discharge scale with the "so-called" Froudian discharge scale as determined by the application of Equation (2*b*)—not Equation (3).

NOTE.—The paper by Kenneth D. Nichols, Jun. Am. Soc. C. E., was published in June, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Herbert W. Ehrgott, Esq.; November, 1938, by E. H. Taylor, Jun. Am. Soc. C. E.; and February, 1939, by Herbert D. Vogel, M. Am. Soc. C. E.

[8] Lieut., Corps of Engrs., U. S. Army; Instr., Dept. of Civ. and Military Eng., U. S. Military Academy, West Point, N. Y.

[8a] Received by the Secretary April 18, 1939.

Captain Ehrgott's remarks on Item No. 7 are particularly appreciated. The additional evidence of the variable results obtained by changing bed materials cannot be repeated too often nor over-emphasized. Complete and careful verification of each and every model is an absolute requirement of sound model technique.

Mr. Taylor's discussion gives the impression that he has accepted a defeatist attitude concerning the use of distorted models. The writer is in agreement that within certain ranges "undistorted models more truly represent their prototypes then distorted models," but as stated in the original paper, for a large prototype very frequently the smallest practical undistorted model is economically unjustifiable from the standpoint of cost whereas a smaller, but yet practical, distorted model may be of definite value and also a good investment economically. A distorted model is not a perfect tool, but it may be better than no tool.

Mr. Taylor's skepticism of verification as being somewhat uncertain is justifiable to a certain extent. Verification may be uncertain and when carelessly and improperly done, it may lead to placing faith in an absolutely untrustworthy model. However, the large number of successful model studies has shown that, in the hands of experienced personnel, a distorted model, properly verified, is of definite value for aiding in the solution of difficult hydraulic problems.

Captain Vogel has presented an excellent discussion of some of the common-sense principles that govern the conception and use of hydraulic models. It is well occasionally to rise above the petty arguments, pro and con, concerning the exact accuracy of hydraulic models and to view the problem as a whole to decide whether or not the results have been of benefit. Moreover, the importance that Captain Vogel attributes to economic considerations cannot be over-emphasized. Practical considerations and experience should outweigh minor theoretical arguments that the tool is imperfect.

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DISCUSSIONS

COST OF ENERGY GENERATION
SECOND SYMPOSIUM ON POWER COSTS

Discussion

BY MESSRS. JOHN C. PAGE, C. F. HIRSHFELD AND R. M. VAN DUZER, JR., H. K. BARROWS, AND MAURICE R. SCHARFF

JOHN C. PAGE,[72] M. AM. SOC. C. E. (by letter).[72a]—The Second Symposium on Power Costs should prove most helpful in clearing away much of the haze that has surrounded some of the questions involved. The problem of computing the "obsolescence" factor, for example, has received particular attention, which has been clarifying. Only a few points raised in discussion require comment.

There seems to be general agreement, among those who participated, on certain fundamentals. These are: (1) That it is impossible to determine, on a general basis, whether steam or water power is cheaper, since each development must be considered separately, taking into consideration physical, economic, and technical facts which vary from place to place and from time to time, the sum total of which may favor either steam or water power in a given instance; and (2) that the electrical industry is monopolistic by nature, and whether privately or publicly operated, must serve in the public interest in self-justification.

Mr. Mason asks who has brought about the advancement in the type and knowledge of equipment for hydrogeneration in the past fifteen years, and answers that it certainly included no one but the private company, the manufacturer, and the private laboratory, thereby excluding all public agencies from any credit for this valuable public service. The record will show that many valuable contributions of such equipment for hydro plants have been made by designers in public employ. Certainly the designing section, Bureau

NOTE.—This Symposium was presented at the meeting of the Power Division, New York, N. Y., January 20, 1938, and published in April, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1938, by Messrs. Joel D. Justin, Frank H. Mason, W. V. Burnell, Daniel W. Mead, A. G. Christie, William E. Rudolph, Louis Elliott, H. G. Gerdes, Leverett S. Lyon, John T. Madden, R. L. Thomas, Theodore B. Parker, Philip Sporn, L. N. McClellan, James C. Bonbright, R. Robinson Rowe, V. B. Libbey, and Edwin D. Dreyfus; November, 1938, by R. H. Parsons, Esq.; and February, 1939, by F. Knapp, Esq.

[72] Commr., U. S. Bureau of Reclamation, Washington, D. C.

[72a] Received by the Secretary April 7, 1939.

of Reclamation, through its laboratory work and otherwise, has improved high-pressure reservoir outlets (to name only one example); and others could be mentioned. Other public agencies also have made important contributions. In the advance of the industry by scientific means, there seems to be sufficient credit for all. In the general advance, there has certainly been no lack of co-operation between engineers and designers (no matter where or by whom employed), and the manufacturers.

Professor Mead states that interest during construction is seldom considered in computing the cost of Federal projects. This statement is true, unless too much emphasis is given to the word "seldom." Interest during construction is not computed on every Federal project. It was computed in the rates for Boulder Dam, to name one, and projects financed by loans from the Public Works Administration carried interest during construction, although the grant which usually accompanied the loan served to offset, in part, at least, the interest charges. The rates at which power is sold by all projects of the Bureau of Reclamation that remain under Federal control are computed to include interest on the capital cost chargeable to power.

In this discussion, Professor Mead intimates that many of the Federal multiple-purpose projects, which include water power, are unwarranted. He indicates that information concerning their cost and their probable returns is not obtainable, and that the conclusion must be reached that they were not adequately planned. The writer calls attention to the contribution by Mr. McClellan, in discussing this Symposium, in which information as to the cost and repayment of the Boulder Dam and Grand Coulee Dam projects is presented. Data similar to these are available on other projects constructed by the Bureau of Reclamation. Although Mr. McClellan's discussion dealt primarily with cost and with returns, the planning and studies affecting these factors were not made more carefully than the investigations of other phases and features of the projects.

There may be instances, as Professor Mead hints, in which the desire to create work on useful projects outweighed other considerations as to desirability, necessity, or soundness, but the writer knows definitely of no such project. Those constructed by the Bureau of Reclamation (and the writer is qualified to speak authoritatively only for the Bureau) may have been begun quickly at the outset of the work programs; but that is no indication that thorough investigation had not preceded them. They were all carefully planned in advance, and some of them had been awaiting construction for long periods. Whether the conclusions upon which the plans were adopted were erroneous, only some historian in the future can say. The preparatory studies, however, were the best that could be made by able and qualified engineers at the time. The writer does not believe that any of these irrigation-power projects will be proved later to be unwarranted expenditures.

C .F. HIRSHFELD,[73] ESQ., AND R. M. VAN DUZER, JR.,[74] ESQ. (by letter).[74a]—Both Messrs. Christie and Elliott have reiterated a fact which the writers have

[73] Chf. of Research, The Detroit Edison Co., Detroit, Mich.
[74] Engr., The Detroit Edison Co., Detroit, Mich.
[74a] Received by the Secretary February 14, 1939.

emphasized—that judgment must be used in arriving at power costs from the data presented in this paper and others comprising the Symposium. The concrete examples contained in Mr. Christie's discussion, showing how the initial plant cost can vary widely with different local conditions, serves further to illustrate that generalizations as to power costs cannot be made.

The writers made no direct mention of reserve equipment and its effect on power costs because of the complications that would have been introduced. The subject could very well serve as the topic for a complete paper. The fact that reserve equipment is necessary for proper maintenance of service is one of the important reasons why few fuel-burning plants show annual capacity factors much in excess of 60 to 70 per cent.

It is unfortunate that the various ratios or factors used to express the significant characteristics of load have never been clearly defined by a recognized engineering body. The writers have observed in several of the papers and discussions that the same terms are used by the several writers with totally different meanings implied. For instance, the terms load factor, capacity factor, and use factor are ratios expressing entirely different characteristics of load. This fact should be recognized when comparing power-cost data.

H. K. BARROWS,[75] M. AM. SOC. C. E. (by letter).[75a]—Mr. Mason's reference to over-capacity of plants recalls an unusual example of this condition in the Cobble Mountain plant of the City of Springfield, Mass., which is one of the features of their Little River water-supply system. This plant develops power from water for municipal supply as well as surplus water now available. Power use is during the eight hours of the day, at such times as needed for peak use in the power system of the Turners Falls Power Company, the lessees of this plant. To adjust conditions of flow to the 24-hr municipal supply needs, there is an intervening balancing reservoir between the power plant and the municipal intake.

The drainage area available is 45.5 sq miles and a net head of 330 to 420 ft, with complete regulation by storage. The wheel capacity is 33 000 kw under the maximum head of 420 ft, which corresponds to 1 160 cu ft per sec and about 25 cu ft per sec per sq mile of wheel capacity. As far as the writer is informed this is the highest flow per square mile of wheel capacity of any plant in the world, corresponding as it does to what would be a flood flow of this river. The yearly output is about 20 000 000 kw-hr, which gives the low capacity factor of about 0.09.

This plant is an example of what might be called a "super-peak load" plant —used only when most needed to help carry seasonal peaks in the load system. It cost about $4 200 000 including dam and reservoir, or $140 per kw, on the basis of 30 000-kw capacity. The power-plant alone, including penstock, surge tank, power-house, and equipment, cost $1 100 000 or $37 per kw.

Mr. Parker calls attention to the effect of long conduits as tending to increase the cost of high-head plants. As he notes, this tends to make the waterway for such plants a large proportion of the investment. On the other hand,

[75] Prof. of Hydr. Eng., Mass. Inst. Tech., and Cons. Engr., Boston, Mass.

[75a] Received by the Secretary February 23, 1939.

this excess cost is often compensated by a smaller cost of dam, as well as of power-house and equipment, as will be noted by reference to Table 7.

The capacity of waterway is fundamentally based upon loss of head under average operating conditions; that is, wheel capacity times capacity factor, keeping in mind that true average losses should be used and not the loss corresponding to average flow or use of water. A low capacity factor, with a long penstock, however, may mean a somewhat greater carrying capacity in order not to lessen, too greatly, the maximum plant output.

For wooden pipes or flumes a greater allowance for depreciation should undoubtedly be made. Of course, it was impracticable for the author to vary this factor in individual cases for lack of detailed information as to structures.

Mr. McClellan has contributed an interesting diagram (Fig. 34) of costs and tentative financing for the Grand Coulee Project on the Columbia River.

Mr. Rowe has called attention to the water-power resources of Southeastern Alaska, where, evidently, conditions are favorable to development, as compared to the leading States in this respect.

The author appreciates the interest shown in his paper and hopes that it may be of some service in the field of water power.

MAURICE R. SCHARFF,[76] M. AM. SOC. C. E. (by letter).[76a]—Professor Bonbright has raised some most interesting questions. The writer agrees with him that it is a common error, in making economic comparisons of comparable installations, to disregard the relative probability of the development of obsolescence in different types of equipment and its resultant effect of probable frequency of replacement. The specific problem which he submits, of the correct comparison of a hydro plant to be scrapped in 50 yr and a steam plant to become obsolete in 20 yr, is an intriguing one. The writer suggests as a theoretical solution that the comparison should be made on the basis of the sum of the present worths of the total costs of the alternate projects, including capital costs and operating costs, for a period of time equal to the least common multiple of the assumed periods of replacement, or 100 yr. The writer agrees fully with Professor Bonbright as to the impossibility of anticipating the developments in steam power generation at the end of periods of 20, 40, 60, and 80 yr, respectively. Such an impossibility, however, does not prevent the submission of a suggestion as to the theoretically correct basis of comparison.

Mr. Burnell and Mr. Dreyfus have called attention helpfully to some of the practical management, and financial, considerations that must inevitably affect the practical determination of depreciation policies. The writer believes, however, that it will be most helpful for the engineering profession to continue the discussion and clarification of the engineering and economic aspects of the problem, leaving these practical financial and management considerations for application to the final engineering conclusions.

Mr. Thomas has emphasized the practical advantages of the application of the sinking fund, or compound interest method of treating depreciation to a large hydro-electric project. The writer has not intended to express any substantial disagreement with this point of view.

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[76] Cons. Engr., New York, N. Y.

[76a] Received by the Secretary April 17, 1939.

Mr. Libbey apparently agrees fully with the writer in his suggestions as to the relationship between annual and accrued depreciation and the equality between the value of the capital investment and the present worth of its amortization, plus the return upon the unamortized part of the investment. He seems to the writer, however, to state the conclusion from this premise in reverse when he suggests that the method chosen for amortization may be conceived of as determining loss of value and thus depreciation. It would seem more correct to the writer to say that the loss of value or depreciation should determine the method chosen for amortization.

In conclusion, the writer is gratified by the interest that has been shown in the subject and expresses the hope that engineers may continue to study the engineering and economic aspects of this difficult problem.

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DISCUSSIONS

A THEORY OF SILT TRANSPORTATION

Discussion

BY C. C. INGLIS, ESQ.

C. C. INGLIS, ESQ.[61] (by letter).[61a]—This paper may be divided into two parts: (1) That dealing with quantity of silt transported; and (2) the theory on which the formulas are based.

The part dealing with quantity of silt transported is interesting and likely to be valuable in many ways; but "grade of silt" is normally more important than "quantity of silt being transported," because it is an independent variable, whereas silt charge is a dependent variable, varying with V, d, and time. For this reason, grade cannot be ignored and the coefficient C_1 must be a variable. The effect of temporal changes in velocity must also be taken into account. In the case of the River Indus, silt charge depends on conditions during the previous eight days, during which the silt charge continues to increase on a rising flood. Similarly, on a falling flood the heavy silt charge tends to persist, and all of the excess silt will not deposit for eight days. This effect is shown clearly in Table 4.

The part of the paper dealing with the theory on which the author's formulas are based is not nearly so "happy." In this paper, as in his earlier paper[7] based on Kennedy's formula

$$V = C\,0.84\,d^{0.64} \qquad (35)$$

the author has assumed that there is a simple, experimental, fundamental formula, applicable to the relation between depth and velocity on all verticals across a channel section. This, definitely, is not the case. As a rough approximation, it is a valuable aid, which has been applied widely in India for many years; but since Mr. Gerald Lacey wrote his first paper in 1928 on "Stable Channels in Alluvium"[62] it has been clear that the Kennedy formula

NOTE.—The paper by W. M. Griffith, Esq., was published in May, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Joe W. Johnson, Jun. Am. Soc. C. E.; October, 1938, by Messrs. George W. Howard, Harry F. Blaney, and E. W. Lane; December, 1938, by Messrs. O. A. Faris, J. E. Christiansen, Samuel Shulits, and Gerald Lacey; February, 1939, by Messrs. Glenn W. Holmes, and Hunter Rouse; and March, 1939, by S. G. Bauer, Esq.

[61] Director, Central Irrigation and Hydrodynamic Research Station, Poona, India.

[61a] Received by the Secretary January 18, 1939.

[7] *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), p. 279.

[62] *Loc. cit.*, Vol. 229 (1929–30), p. 259.

was only an approximation, and that velocity and dimensions of channels were related, simply and directly, to hydraulic mean depth, and only indirectly to depth.

The main objection to the Kennedy formula has been that it ignored shape—that is, according to the formula, a channel with a normal silt value expressed by Equation (35) might be 1 000 ft wide and 1 ft deep or 23 ft wide and 10 ft deep.

The author has evolved a method of determining "the section having the maximum silt carrying capacity" by trial and error from his approximate formulas. This shows that he too considers that the shape of a channel may be varied. Where the material is inerodible this is true; but in erodible material, especially where the silt charge is considerable, the channel (as pointed out by E. S. Lindley, M. Am. Soc. C. E., in 1919[63]) is uniquely determined. Mr. Lacey produced the following formulas giving dimensions for regime channels in 1928:[62]

$$P = 2.668\, Q^{1/2} \quad \ldots\ldots (36)$$

$$V = 1.17\, \sqrt{f\, R} \quad \ldots\ldots (37)$$

$$V = \left(\frac{Q\, f^2}{3.8}\right)^{1/6} \quad \ldots\ldots (38)$$

$$R = \frac{0.7305\, V^{1/2}}{f} \quad \ldots\ldots (39)$$

and,

$$S = \frac{f^{5/2}}{2\,587\, Q^{1/9}} \quad \ldots\ldots (40)$$

in which f was a silt factor depending on silt grade, but affected by silt charge. From Equations (38) and (39)

$$R = 0.468\left(\frac{Q}{f}\right)^{1/3} \quad \ldots\ldots (41)$$

Mr. Lacey wrote his second paper "Uniform Flow in Alluvial Rivers and Canals"[64] in 1933 and since then he has improved his formulas dimensionally

TABLE 11.—Comparison of Formulas.

Lacey	Punjab Irrigation Research Institute
$P = 2.667\, Q^{1/2}$	$P = 2.80\, Q^{1/2}$
$R = 0.473\left(\frac{Q}{f}\right)^{1/3}$	$R = 0.47\, Q^{1/3}$
$V = 1.1512\, (f\, R)^{1/2}$	$V = 1.11\, R^{1/2}$
$S = \frac{f^{5/3}}{1\,858.6\, Q^{1/6}}$	.....

by introducing kinematic viscosity and gravity. Subsequently he modified the exponents of his original, simple formulas, very slightly. It is interesting to compare these results with the formulas derived statistically by the Punjab

[63] "Regime Channels," *Proceedings*, Punjab Eng. Congress, Vol. VII, 1919.
[64] *Minutes of Proceedings*, Inst. C. E., Vol. 237 (1933–34), Pt. 1, p. 421.

Irrigation Research Institute from a mass of Punjab data for silts of less than 0.6 mm diameter (see Table 11). The principal difference in the Punjab formulas is the omission of any silt factor. This does not mean, of course, that a silt factor should not enter the formulas—silt must affect the last three formulas and in the writer's opinion also the P–Q relationship—but merely that owing to the small range of silt found in the Punjab Canals and the low powers in which the silt factor enters the equation, it has not been possible to obtain a significant correlation.

The Punjab Irrigation Research Institute have never accepted the Lacey formulas and have produced formulas of their own, which, until recently, have differed considerably from the Lacey formulas. The fact that they have now produced almost similar formulas, after five years of exhaustive and painstaking work, shows that the regime relationships derived by Mr. Lacey are fundamental. They are not, however, directly applicable to variable, non-regime conditions; and it is for non-regime conditions that approximate formulas of the type evolved by the author are valuable.

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DISCUSSIONS

AERODYNAMICS OF THE PERISPHERE AND TRYLON AT WORLD'S FAIR

Discussion

BY W. B. KLEMPERER, ESQ.

W. B. KLEMPERER,[24] ESQ. (by letter).[24a]—The subject of the paper is of interest to the aeronautical engineer especially because of its close analogy with the problems arising from the groundhandling of large free balloons. Every opportunity to supplement theoretical investigations with experimental research, therefore, is keenly appreciated.

The approximate theoretical treatment of the Perisphere in representing the effect of a mirror image sphere by a mere increase of the axial doublet strength, as adopted in the paper, is a close approximation only where the ground distance is large compared to the radius. However, the closer the ground distance the poorer the approximation becomes. It neglects the flow constraint near the ground and leaves no room for any vertical force acting on the sphere without pedestals.

With the ground clearance only a small fraction of the radius, the first-order mirror doublet almost doubles the velocity increment at the bottom of the sphere, whereas it contributes only about 4% at the top. The body generated by a space doublet does not stay spherical when the flow of a parallel mirror doublet in its proximity is added to a general parallel flow.

Therefore, the observed vertical nonsymmetry of the pressure distribution need not be interpreted as a discrepancy between theory and experiment. Thus the high suction observed in the bottom region may not all be due to the ground board's boundary layer restricting the passage, as was suggested in Conclusion (2), but also to a genuine Venturi effect which can be embraced by a classical theory taking the nonsymmetry of the mirror-flow system with respect to the sphere center level into account.

Such a theory requires the introduction of successively mirrored doublets,

NOTE.—The paper by Messrs. Alexander Klemin, Everett B. Schaeffer, and J. G. Beerer, Jr., was published in May, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Messrs. Elliott G. Reid, and Karl Arnstein.

[24] West Los Angeles, Calif.

[24a] Received by the Secretary June 16, 1938.

or sources and sinks, eccentrically located in the spheres. Lamb postulates an attraction between the sphere and the ground, for the ideal fluid.[25]

The fact, however, remains that as soon as circulation is created about the sphere by ground surface friction, etc., a lift force is generated which may outweigh the Venturi attraction. The same effect may be caused by a large-scale wind boundary layer extending high above the sphere, much larger than would correspond to the relatively thin boundary layer adjoining the ground board in the wind tunnel. The real sphere, therefore, may have a greater lift than the model. This additional lift could be expected to be essentially proportional to the average vertical wind gradient throughout a layer comparable in thickness to the height of the structure.

The question of what pressure will actually prevail inside the spherical building would appear to be worth while to answer by full-size experiment. The tendency to keep out draft favors leeward ventilation and most likely works toward lowering the interior pressure below atmospheric static. An exposition building of this kind might lend itself to the installation of recording gages.

[25] "Hydrodynamics," by H. Lamb, Fifth Edition, pp. 124, 174.

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DISCUSSIONS

LATERAL EARTH AND CONCRETE PRESSURES

Discussion

BY RALPH W. STEWART, M. AM. SOC. C. E.

RALPH W. STEWART,[45] M. AM. SOC. C. E. (by letter).[45a]—Lateral pressures on retaining walls produced by concentrated loads on the ground surface behind the walls, which is investigated by the authors, was also studied both experimentally and analytically by M. G. Spangler, Assoc. M. Am. Soc. C. E., at the Iowa Engineering Experiment Station and the results were published in April, 1938.[46] One rather important difference is noted between the authors' conception of the distribution of pressure and the findings by Professor Spangler. In Fig. 2 the lateral pressure due to a surface load which is set back some distance from the back of the wall is indicated as maximum at the top of the wall. The Iowa tests show this pressure as zero. The latter is obviously correct as a thin stratum of granular earth along the surface could not act as a strut to transmit horizontal pressure from *P* (in Fig. 2) to the top of the wall. If called upon to perform this duty it would fail by buckling—that is, the surface of the ground would bulge upward. As illustrated in Fig. 15, the actual pressure distribution curve, according to the numerous observations reported in Iowa, forms a bulb whose maximum ordinate is somewhat below the top of the wall, depending on how far back the load is.

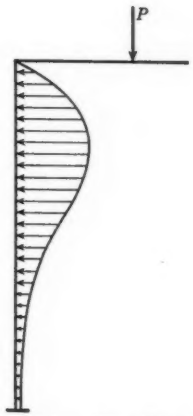

FIG. 15.—TYPICAL DISTRIBUTION OF LATERAL PRESSURES AGAINST A VERTICAL FACE DUE TO A SURFACE LOAD

NOTE.—The paper by Lararus White and George Paaswell, Members, Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by A. E. Cummings, M. Am. Soc. C. E.; December, 1938, by Messrs. Robert G. Hennes, Robert F. Legget, and Charles Terzaghi; January, 1939, by Messrs. Jacob Feld, M. G. Spangler, and Raymond D. Mindlin; February, 1939, by D. P. Krynine, M. Am. Soc. C. E.; and March, 1939, by David A. Molitor, M. Am. Soc. C. E.

[45] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

[45a] Received by the Secretary April 3, 1939.

[46] Iowa State College Bulletin No. 140, Ames, Iowa, 1938.

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DISCUSSIONS

WIND FORCES ON A TALL BUILDING

Discussion

BY OTTO GOTTSCHALK, ESQ.

OTTO GOTTSCHALK,[48] ESQ. (by letter).[48a]—In presenting field-tests data pertaining to the effect of masonry on a structural steel building frame, the author has certainly stirred up a desire to learn similar facts about as many existing tall buildings as possible. The opportunity to compare the behavior of a building with a true model of its skeleton will not occur frequently. This suggests that there should be more simple methods of determining the same information.

In 1938,[49] the writer presented a simple method of computing wind stresses (especially column pressures) from a general model analysis. If the column pressures are known, the distribution of the stress between various columns can be computed by this method. They can be computed, furthermore, from the shortening or lengthening of the columns in a building and, approximately, from the difference in elevation between corresponding points on adjacent columns.[49] It should be easy and inexpensive to weld special bolts or sockets to the field frame at several floors of existing buildings. Since the period of vibration of tall buildings is always equal to several seconds, it will be possible to obtain the change of level between both by a simple device such as a water-level which can be adjusted to permit accurate readings to within 0.002 in. Such readings will supply extremely valuable information as applied to columns that are either isolated or embedded in walls or partitions, and also would offer a means of appraising the effect of the masonry covering.

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by Messrs. David C. Coyle, and Clyde T. Morris; January, 1939, by Messrs. Robins Fleming, F. P. Shearwood, Lydik S. Jacobsen, Francis L. Castleman, Jr., J. B. Wilbur, R. D. Spellman, David A. Molitor, Walter J. Gray, and K. L. DeBlois; and March, 1939, by Messrs. Albert Smith, and Victor R. Bergman.

[48] Buenos Aires, Argentine Republic.

[48a] Received by the Secretary March 30, 1939.

[49] "Simplified Wind-Stress Analysis of Tall Buildings," by Otto Gottschalk, *Proceedings*, Am. Soc. C. E., December, 1938, p. 2023.

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DISCUSSIONS

SETTLEMENT STUDIES OF STRUCTURES IN EGYPT

Discussion

BY W. S. HANNA, ESQ.

W. S. HANNA,[16] ESQ. (by letter).[16a]—The method applied to Buildings I, II, and III in this paper gave a fairly satisfactory forecast of settlements, based on laboratory tests (see heading, "Results of the Settlement Studies: Relation Between Settlements in Clays as Actually Observed and as Forecast on the Basis of Laboratory Tests"). Briefly, the process involves loading the sample until it has reached its preconsolidation pressure, and then unloading it until it sustains only the weight of the upper layers. The resulting branch of the compression curve is taken as the basis for computing the calculated settlement forecast.

Buildings investigated subsequently by the Laboratory at Egyptian University gave different results; in a few cases the predicted settlements were less than the actual settlements; whereas in the cases reported by the author the forecast settlements were higher than the actual. Professor Tschebotareff tried several methods, of which the foregoing seemed to fit most closely. Contradictory results observed subsequently led the writer to investigate the entire problem again. There is no doubt that the samples of brown clay expand while being removed. On the other hand, it seems improbable that the resulting strains can be avoided by the process described in the paper, which amounts to remaking the sample in the odometer until a state similar to its condition in the soil is reached. This procedure ignores all the plastic strains produced and the effect of confining the sample in the odometer. Furthermore, there is considerable difference in the compression curve of undisturbed local clays and the curves derived from clay samples, and this is observed in both the swelling and the non-swelling types. Therefore, the writer undertook to study the entire problem from a new angle. Preliminary results of these studies indicate

NOTE.—This paper by Gregory P. Tschebotareff, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. L. C. Wilcoxen, Trent R. Dames, and Edwin J. Beugler.

[16] Head of Soil Mechanics and Foundation Research Laboratory, Egyptian Univ., Cairo, Egypt.

[16a] Received by the Secretary February 10, 1939.

that the author's statement that the method described in the paper has proved a success cannot be considered final.

The writer does not agree with Professor Tschebotareff's conclusions regarding the pronounced effect on some time-settlement curves of the annual rise and fall of ground-water level as a result of the flood. He states (see heading "Results of the Settlement Studies: Ground-Water Level") that "A pronounced effect on the time-settlement curve [due to the variation of the subsoil water level during the yearly flood season] was observed only when the rise of the ground-water level coincided with the period of construction and when the seat of settlement was located in a deep, dark, clay layer" (words in brackets are by the writer). On the contrary, the writer believes that the effect is mainly due to the upper layers traversed by the rising water, which consists generally of the brown clay. The reasons are briefly as follows:

(*a*) The decrease of pressure on the deep, dark clay, caused by the buoyancy of the upper layers affected by the rising water, is quite small. It is true that buoyancy decreases the specific weight of the layer through which the water flows; but the specific weight is increased as a result of the increase in the water content. The actual weight of 1 cu m of brown clay, below normal ground-water level, is 1.65 tons; but it weighs 2.08 tons during the flood season, when weighed after it has been disturbed. Except in its undisturbed position in the immediate vicinity of the river, the maximum rise is 2 m. In such cases, therefore, the maximum reduction in pressure as a result of buoyancy is $2 \times 0.057 = 0.114$ kg per sq cm (233 lb per sq ft). When part of the period of construction coincides with the flood period, the increase in pressure due to the weight of the buildings ranges from 0.4 to 0.8 kg per sq cm of the area of the building (817 to 1 635 lb per sq ft). Of course, these values do not represent the component of the increased pressure on the deep layers, but they furnish a basis of comparison. In computing the buoyancy, furthermore, full upward hydrostatic pressure has been assumed, which is impossible in the case of plastic clay layers.

(*b*) In most cases the layers through which the flood water passes are the brown clays, which are either stiff or soft and silty. The stiff clay, if permeable, exhibits strong swelling tendencies when placed in the odometer. This is not elastic swelling, because the sample has had sufficient freedom to swell completely during the shaping and after leaving the sampler. It is due to the absorption of water from the porous stone when it is placed in the odometer.

The writer conducted several tests to study the effect of the swelling due to absorption. The pressures required to return the 12-mm sample to normal, when swelling occurs under the weight of the piston alone, vary between 0.18 and 0.30 kg per sq cm (368 and 613 lb per sq ft, respectively). Therefore, it is safe to assume that, where such brown clays are reached by rising water, the swelling of these layers reduces the pressure on the layer above them and, probably, to a slight degree, on the lower layers (this is a mere assumption). When the water recedes and the soil retains its normal water content, the swelling resistance disappears and the resultant pressure on the upper layers is

suddenly increased. This explains the reduced rate of the time-settlement curve during the rise, and the increased rate, during the fall, of water level. Furthermore, it is a known fact that most old houses settle soon after the ground-water level beneath them has been lowered. The slight movement of the building when it responds to the swelling of the soil seems sufficient to crack its walls. Most of these houses have bearing pressures not exceeding 0.3 kg per sq cm (613 lb per sq ft) and the mortar, which was originally of a poor quality, has deteriorated with age.

As compared with dark clays the brown clays are relatively little affected by remolding when tested in the unconfined state (see heading "Results of the Settlement Studies: Not All Clays Are Unfavorably Affected by Remolding"); and brown clays, of the same origin as those found in the interior, have been found near the sea. This brown clay is very remoldable. Furthermore, extremely silty types of it, which are very stiff in the natural state, crumble under the smallest pressure when remolded; and, both the brown and dark clays are strongly affected by remolding when tested in the confined state, a case which is not observed in samples of brown and dark clays found in the interior. It would be quite interesting, therefore, to learn whether similar behavior in clays deposited under saline water has been observed elsewhere.

The writer agrees entirely with the author's statement (see "Results of the Settlement Studies: Efficiency of Devices for Taking 'Undisturbed Samplers' ") that in the swelling types of clays, uninterrupted lateral confinement becomes essential for all stages until it is placed in the odometer. Some time ago the writer experimented with a special ring that can be fitted both to the samples and to the odometer, so that the soil sample can be tested without removing it from the sampler. It was tested in connection with the different samplers and also was used alone to extract the sample after the hole was cleared directly under the bore tubes. Some interesting, and some rather contradictory, results were obtained. On the whole, however, the effect of lateral swelling seemed to be about 100% in the clays most seriously affected. This means that the investigator must search elsewhere for the explanations of the difference (in some cases of more than 100%) between the actual values of settlements and the values based on the odometer tests.

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DISCUSSIONS

ANALYSIS OF RUN-OFF CHARACTERISTICS

Discussion

BY C. O. CLARK, JUN. AM. SOC. C. E.

C. O. CLARK,[24] JUN. AM. SOC. C. E. (by letter).[24a]—In bringing to the attention of hydrologists the alteration of the hydrograph effected by channel storage, Mr. Meyer points to an agent ordinarily considered to be small, but an understanding of which is essential to one striving to avoid misuse of the unit hydrograph, and to reproduce or predict stream flow with greater accuracy.

Any one wishing to make a more extensive investigation into this problem will find the theory expressed in the Muskingum method of flood routing a valuable tool for correlating Mr. Meyer's analysis of the hydrograph with the familiar function of storage in the usual routing of floods.

The writer is familiar with no generally available publication which explains this method, but reference to it, and definition of terms, have been made by T. T. Knappen,[25] M. Am. Soc. C. E. The terminology used by Mr. Knappen is adopted herein, and the formula is not repeated.

Further development of the flood routing equation shows, in addition to the explanation in the aforementioned discussion, that

$$o_2 = C_3\, i_2 + C_4\, i_1 + C_5\, o_1 \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which

$$C_3 = -\frac{K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \ldots\ldots\ldots\ldots\ldots\ldots (27a)$$

$$C_4 = +\frac{K\,x + 0.5\,T}{K - K\,x + 0.5\,T} \ldots\ldots\ldots\ldots\ldots\ldots (27b)$$

and,

$$C_5 = +\frac{K - K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \ldots\ldots\ldots\ldots\ldots\ldots (27c)$$

NOTE.—This paper by Otto H. Meyer, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. Victor H. Cochrane, and Bertram S. Barnes; March, 1939, by Messrs. LeRoy K. Sherman, Richmond T. Zoch, and Merrill Bernard; and April, 1939, by Messrs. Franklin F. Snyder, and W. G. Hoyt.

[24] Asst. Engr., U. S. Engr. Office, Norfolk, Va.

[24a] Received by the Secretary March 6, 1939.

[25] "National Aspects of Flood Control: A Symposium," *Transactions*, Am. Soc. C. E., Vol. 103 (1938), Equation (3), p. 667.

It will be noted that C_5 is the same as the author's "λ," when $x = 0$—that is, when storage is solely a function of discharge. By analyzing the falling hydrograph, one can determine the value of K, which is the differential of the storage-discharge equation, and which, when storage varies directly with discharge, is the author's C_2. Knowing the value of K, and its variation with discharge, one can determine a storage-discharge curve, about which storage studies revolve, regardless of whether the ratio of total storage to instantaneous discharge is a constant or not, as long as the variation of the ratio is uniform. It will be seen that the assumption of direct variation is not necessary, except where the convenience of use outweighs the desirability of accuracy, or where the quality of fundamental data does not warrant the refinement of considering the variation. The writer is of the opinion that the assumption of uniform variation is sufficiently correct on many, and most, streams to warrant calculations on that basis except where reliable hourly records of rainfall are available. The graphical methods in use in routing floods through reservoirs, or reaches where storage is a function of discharge stage, will be found applicable and convenient.

The greatest advantage of studying the hydrograph with this tool may prove to be the indication that the storage in the ground-water table may be correlated with stream discharge stage and thereby enable one to use values of run-off that will be not less than the monthly average run-off, without resort to the somewhat confusing and highly artificial concept of variable base flow. For example, on a particular stream, with a drainage area of about 1 300 sq miles and 75 miles maximum length, the minimum value of "K" on the falling limb of several hydrographs of varying size, and from rainfall of various intensities and duration, appeared to be fairly constant at about 8.0 hr, but below a discharge of 6 cu ft per sec per sq mile the values were greater and had a common trend, but were not strictly uniform. The larger values were approximately 200 hr.

Detailed analysis showed that the variation could be explained and duplicated closely by a compound unit graph, the sum of two simple hydrographs, one of surface run-off and one of sub-surface run-off. The surface run-off, taken as 70% of the total run-off, was assumed to contribute according to the author's histogram, routed through channel storage of 8.0 hr by the Muskingum method. The sub-surface run-off (30% of the total run-off) was taken as contribution by the histogram routed through 200 hr of storage. These two hydrographs, added together, make up a compound unit hydrograph which has all of the advantages that the author claims for his basic hydrograph, and does not need to be used with a variable base flow to secure reasonably accurate and consistent results for the entire hydrograph, using percentages of run-off highly consistent with the monthly average run-off, base flow included. It will be apparent that the unit hydrograph is almost the same as the author's basic hydrograph for surface run-off plus another for sub-surface run-off. As Mr. Meyer states, the bases are infinite, but may be used as constant, of such length as will include 95% to 98% of the volume, without appreciable error.

Confining attention to the constant storage-discharge relation assumed by the author, the writer finds little difference between the unit hydrograph and the author's basic hydrograph, and therefore uses the terms interchangeably. A unit hydrograph that has been routed through such channel storage above the

gage has the infinite base length to which the author refers, may be applied to variable quantities of run-off in successive unit periods of time, and will provide a more accurate indication of discharge than can be justified in the use of the usually available rainfall records. The well-derived unit hydrograph will show the effect of constant-ratio storage and is only less accurate than the author's method in that very small ordinates are conveniently neglected, for the sake of ease in computation, and a constant base length of hydrograph results. The lack of agreement between the new method and the old method in computing the hydrographs of the Merrimac River, shown in Fig. 7, could only result because the unit hydrograph used was not as well derived as the basic hydrograph. Although the writer has not proved that this statement is true algebraically, he has proved it to his satisfaction by routing hydrographs of various shapes through reaches of constant storage-discharge relation to prove that the sum of the separately routed hydrographs is equal to the routed hydrograph of the sum of the separate hydrographs.

The use of the concentration curve and the storage curve to derive the unit or basic hydrograph of an area has very real value. To be consistent, however, it would appear that the rising curve should show the effect of storage, because the same storage which holds the hydrograph up on the falling leg holds it down on the rising leg and accounts for much of the lag between initial rainfall and initial rapid rise of the stream.

The writer would like to raise one question with regard to the use of the histogram. If the contribution of the areas of varying distance from the gage combine to form the concentration curve, does it not necessarily follow that the contribution to the channel must fall off in the same manner? The writer has found hydrographs which exhibit the very rapidly falling leg and the angular break near the peak; but those found resulted from rainfall of several periods of duration which terminated in a short period of extremely intense rainfall, rather than, as the author suggests, from a prolonged rainfall of fairly uniform intensity.

The author's derivation of time of travel and, therefrom, the concentration time of a given area, will be useful in a system of rivers which has followed a fairly uniform geological formation (such as the Mississippi System), because there is a general symmetry within the basin; but this concept must be used with caution in river systems which have developed through much stream piracy, because in these may be found slopes in head-water streams which are less than the slopes in the main stream, near the mouth. This condition is quite common in streams in the South Atlantic Coast areas. Of course, the latter type may not meet the author's criteria for similar streams.

The analysis of stream run-off is susceptible of much more accurate computation than has yet been applied to it; by combining the concept of storage modification and the unit or basic hydrograph, degrees of accuracy in computation may be obtained, for many areas, which are limited at the present time by lack of hourly rainfall records. The increase in the number of such stations, and in recorder stream gages, will enable those who, like the author, are unwilling to accept the accepted methods, to delve into the problem and find the mechanism by which very high degrees of accuracy in discharge forecasting will be possible.

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DISCUSSIONS

LAND SURVEYS AND TITLES

FIRST PROGRESS REPORT OF THE JOINT COMMITTEE OF THE REAL PROPERTY DIVISION, AMERICAN BAR ASSOCIATION AND THE SURVEYING AND MAPPING DIVISION, AMERICAN SOCIETY OF CIVIL ENGINEERS

Discussion

BY MESSRS. J. C. CARPENTER, AND DAVID A. MOLITOR

J. C. CARPENTER,[5] M. AM. SOC. C. E. (by letter).[5a]—This report will give impetus to the campaign to "sell" the advantages of better surveying practice and title procedure to practicing engineers, surveyors, attorneys, abstracters, real estate brokers, and land owners. The task of "selling" will be assumed by a small group, enthusiastic about the value of plane co-ordinate control. This enthusiasm must be communicated to the multitude of surveyors and engineers who will actually use the control and by them to the other agencies mentioned herein. There are several other organizations that can aid in the work being sponsored by this joint committee. Abstracter and title associations, real estate dealers' organizations, and all other associations that are concerned in the transfer and use of real estate should be enlisted. It would seem that the best approach to the solution of this co-operative effort should be through the local groups of the national organizations. It will be difficult to convince the legal profession that their business will not decrease when the descriptions of property have been stabilized so that there will be no disputes over boundaries. Certainly the land-owning public will have greater respect for the legal and engineering professions when they find that these boundaries can be so definitely fixed that there can be no dispute about them; and the quality of service will improve if the quantity does not increase. Abstracters and real estate brokers

NOTE.—This First Progress Report of the Joint Committee of the Real Property Division, American Bar Association, and the Surveying and Mapping Division, American Society of Civil Engineers, on Land Surveys and Titles, was published in November, 1938, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: January, 1939, by William Bowie, M. Am. Soc. C. E.; and March, 1939, by Messrs. Joseph C. Federick and Wayne D. Heydecker, and Earl F. Church.

[5] Senior Highway Engr., U. S. Bureau of Public Roads, Fort Worth, Tex.

[5a] Received by the Secretary January 25, 1939.

will be interested in a system that stabilizes their title work and reduces the expense of their operations. Owners of land must be vitally interested in the perfection of a scientific, orderly, and business-like method of boundary description, and the legal perpetuation of these boundaries, thus established. First, however, engineers and surveyors who are intimately connected with the subdivision, surveying, and re-surveying of land, must be "sold" on the method of plane co-ordinate control; and they, in turn, must "sell" these other agencies the merits of the process.

There is every argument for, and none against, the adoption of the universal use of the "State System of Plane Co-ordinates" as a method of defining the positions of inter-related monuments. The procedure has been thoroughly tested and found to be scientifically sound, practicable, and simple in application. The control network has so many other applications, such as for mapping, accurate surface and sub-surface surveys, all kinds of planning, etc., that there is no question as to its eventual adoption as the rigid framework for engineering operations, and the sooner it is adopted the greater will be the public gain.

A. H. Holt, M. Am. Soc. C. E., sets forth[6] the cardinal principle that, "A boundary line having once been authoritatively—even though inaccurately, or even erroneously—fixed in position on the ground, the true purpose of a resurvey is to reproduce that location; not to 'correct,' or otherwise change, it." Professor Holt states, in reference to plane co-ordinates, that:

> "To attempt to substitute their use abruptly for time-tried methods of stating the position of property corners would very likely result in encountering such resistance as would defeat the purpose. A metes and bounds description of land can easily be so written that all the advantages of the use of the coordinate will be secured, while, at the same time, it would be possible to delete all reference to coordinates or coordinate system and still leave a description that would be entirely acceptable under present standards. This latter feature might make such a description acceptable to abstractors, lawyers and courts, in the absence of a formal legalizing act."

Professor Holt also gives an example of a description of this type. This policy might well be incorporated in the recommendations of the Joint Committee for the sole purpose of winning some of the "die-hard" surveyors, abstracters, lawyers, and courts to the cause of the use of plane co-ordinates. Even after a legalizing act is passed, it will be necessary to use the present method of description, paralleled by the co-ordinate description, in all legal instruments, until the co-ordinate system of description becomes so generally recognized that the old method may be abandoned. Once the simplicity and value of the co-ordinate system is demonstrated, there is no doubt that the old system will be forgotten.

Survey work in the Mid-Western section of the United States always passes through three rather distinct stages: (1) The exploratory, or large subdivision stage, which covers the opening and marking off of the large areas for settlement and is characterized by the use of comparatively crude methods and the placing of more or less temporary markers, such as a stake, mound, and two or four pits;

[6] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1158.

(2) the re-survey stage after settlement when the land became more valuable (this work entails much more accurate methods of measurement, and comparatively permanent markers of stone or steel are placed); and (3) the plane co-ordinate system of control, which ties the boundaries previously set out in the two preceding stages to the parallels of latitude and the meridians of longitude, and stabilizes the entire procedure. It is the writer's opinion that this last stage will be the final one. There has been very little use of this stage; but it is certain of development because it is the logical and business-like method of carrying on this important engineering and land-use function; and the horizontal control is being projected across the United States to such an extent that it can be used.

In the "public land States," the Government subdivision of land was the first stage of the survey. This was in reality only an indexing procedure that assigned numbers to regularly shaped tracts and marked the corners so that the "home-seekers" could find the quarter sections. The courses were run with compass and link chain and the markers were usually of the "sketchy" type. One of the "mound builders" who had worked on an original survey was a chainman on a re-survey party under the writer's supervision in South Dakota. He related that he was employed to drive charred stakes and build mounds and pits, and that he and his partner had a light "democrat" wagon for carrying their stakes and tools. They followed the trail of the chainmen through the tall prairie grass and one of them counted the revolutions of the wheel with a rag tied to a spoke, so that they could locate the knot tied in the grass at the point where they were to build a corner. This may have been the origin of the erroneous story that a large part of the western country was surveyed with a wagon wheel. If an antelope distracted their attention the "mound builders" might lose the count and fail to find the knot. Sometimes the corners were placed without finding the knot. If they ran out of stakes before the day was finished, they used the charred match, which had lighted a pipe, as the "charred stake" they certified had been driven at the location.

After the public land States were settled, farm operations, road construction and other developments destroyed many of the corners. When a surveyor is called to re-establish a lost corner, he must attempt to locate the position of the original mark; and the successful accomplishment of this endeavor means the exercise of considerable skill and good judgment, a knowledge of the country and its customs, and some excavation. Failing to locate any part of the original mark, there are instances in which the corner has been re-established by reliable evidence of its position, as determined by surrounding landmarks of unquestioned position. All other methods having been utilized without a final solution, the surveyor must resort to the instructions in the manual of the U. S. Land Office for the "Restoration of Lost or Obliterated Corners and Subdivision of Sections." If luck is with him and the location as found by this last resort does not vary too much from the original point as recognized by the "old settlers," the marker becomes established; and after a reasonable time it has all the authority of the original corner. Luck failing the surveyor, and the point established being in a plowed field remote from the land lines in use, the

case may be taken to Court and the surveyor's measurements nullified by testimony of competent witnesses, or they may be confirmed and allowed to stand. There is always the possibility that the case may be reopened and the boundary questioned and changed.

In some cases the original surveys have been retraced over a considerable area and substantial monuments placed. In very few instances these re-established boundaries have been tied into the geodetic network. This is the second stage described herein, where dependence is placed upon the stability of a monument of stone, concrete or some other hard and heavy material to provide stability for the locations of the corners. The third and final stage will be completed when these monuments are tied into the plane co-ordinates which are the simple arithmetic expressions of the spherical latitudes between the equator and the poles and the longitudes referenced to the zero on the meridian through Greenwich.

The history of land surveys in Texas took a different course from that of the "Public Land States." Discovered by Spanish explorers early in the Sixteenth Century, Texas was included in the Spanish province of Mexico until 1821, and from that date to independence in 1836 was a part of the Republic. The Spaniards and Mexicans distributed the land with lavish hand to colonists and other impresarios, among whom was Stephen F. Austin. The winning of independence and admission to the Union in 1846 advertised Texas and brought about active trade and speculation in her lands. Large areas were granted to individuals as rewards for service. Many acres were sold at 50 cents per acre. When Texas came into the Union it was agreed that her landed estate should remain separate property. The State gave more than 32 million acres to the railroads to promote the construction of a transportation system. When the Constitution of 1876 was adopted it gave 52 million acres to the Public Free School Fund.

This brief outline of the history of land surveys in Texas indicates the methods used in the partition of more than a quarter of a million square miles of territory. An important element of the present situation in regard to boundaries must be obvious from a review of this history. It indicates that at the time of the surveys land values were exceedingly small, there was no demand for accuracy, and the methods of surveying as well as the available instruments, such as the Jacob Staff compass and the link chain, were crude and not adapted to the requirements of accurate subdivision of this large area. Hostile Indians harried the survey parties and this was another element in the development of a practice of establishing a few monuments and projecting a subdivision in the comparative safety of the land office.[7]

Numerous re-surveys have been made with increasingly higher standards of accuracy as the lands became more valuable. The discovery of oil in many sections has increased the values to fabulous figures entirely out of keeping with the uncertainty as to the location of the boundaries. In the re-surveys there has been an attempt to re-establish the locations as marked by the original surveyor and to perpetuate them through the establishment of more or less

[7] See *The Texas Engineer*, Official Publication, Texas Section, Am. Soc. C. E., Centennial Edition, December, 1936.

permanent markers. In some cases the work has been tied in to the geodetic network, but no universal adoption of the plane co-ordinate control has been instituted. Several land divisions of major oil companies and some few independent surveyors have interested themselves in the use of the plane co-ordinate system. Many cases of overlap and vacancy have been discovered and there has recently developed a psuedo-legitimate practice known as "vacancy hunting." Practitioners search the records and, under existing law, the finder has a prior right to buy the land from the State. This practice tends to cloud the title to adjacent areas, and a strip a few feet wide and a mile or more long may become extremely valuable. Attempts are being made to correct this situation by legislative action to control the establishment of title to the vacancy. Legislation adopting the method of geodetic control as advocated in this joint committee report is the logical method of correction of the trouble Co-ordinate control will eliminate all possibility of overlap or vacancy, will correctly set forth the actual area of all tracts, and will clarify and standardize the title procedure for the State. It is the "made to order" solution of the problem and will ultimately be adopted.

In addition to the advantages as a method of title procedure, the distribution of geodetic control will provide an enormously valuable, accurate network for modern mapping and all other engineering works. This value is becoming more and more evident, and the recognition of it will aid in the earlier adoption of the co-ordinate control.

Some changes in the suggested Model Act are required. Under Section 1, the boundaries of zones must be described in such a manner that they will not change when county boundaries are rearranged. The text for the last two paragraphs should be changed to read, "The area included in the following counties, as constituted at the time of the passage of this act, shall constitute the * * * zone." Although the word "now" as included in the Model Act may be legally sufficient, the wording as suggested by the writer leaves no doubt as to the boundaries, and serves to emphasize the fact that these county boundaries are used as a convenient method of delineating the boundaries of the zones as established by the United States Coast and Geodetic Survey.

Under Section 5, the origin of x co-ordinates is indicated to be at the intersection of the central meridian and the parallel 41° 31′ N. This statement is not strictly true because in these computations the origin is considered to be at an arbitrary point to the left of the central meridian such that the x co-ordinate of the central meridian will have a value of 2 000 000. It makes no difference where this arbitrary point is located, but it is essential that all x co-ordinates have a positive value. It is also essential that the Act be absolutely clear and positive in the description of the method of control, for if a case were brought to Court and the meaning of the word "origin" became a point of issue, any vagueness or uncertainty will lose the case for the co-ordinates. It is suggested that the last two sentences of the second paragraph of Section 5 be changed to read "The origin of co-ordinates for this zone is so located that the intersection of the meridian 93°30′ west longitude and the parallel 41°31′ north latitude shall have the co-ordinates $x = 2\,000\,000$ ft; and $y = 0$ ft." A corresponding change should be made in the third paragraph.

Section 6 of the Model Act appears to limit the use of the descriptions to those within a radius of one-half mile of a triangulation station. This makes the act practically inoperative until triangulation stations are established at spacings of about one mile. In the areas covered by the older triangulation the stations are located on the high points of the topography 50 or more miles apart, and there is no reason to expect that important land boundary monuments will be situated within a half mile of these points. This requirement of the Model Act is written in for the purpose of providing a limitation that will protect it from misuse unless and until provision is made for the administration of the act by some authority which will prescribe standards of surveying practice and accuracy to meet the requirements indicated to be desirable. This requirement can, and will, discourage the adoption of the co-ordinate method of control because it will mean an exceedingly limited use of the system. It seems desirable to provide definitely for the establishment of a control authority in the suggested Model Act. This may discourage the passage of the act in some States because there is a widespread sentiment among legislators against the setting up of any additional commissions or boards; but, if this procedure is necessary for the successful administration of the act, it should be included. Postponement of the enactment of an act that will insure satisfactory administration is preferable to early enactment of an act that will be vulnerable to attack and may be unworkable in practice.

Any act prepared for submission to the legislature should be referred to the United States Coast and Geodetic Survey. The Survey has made itself responsible for the technical description of the zones and the formulation of policies involved in the use of this proposed method. It is vital to the successful working of the legislation that all of its features be reviewed by this responsible agency and that no change be made without first obtaining the approval of the Survey.

David A. Molitor,[8] M. Am. Soc. C. E. (by letter).[8a]—The importance of fixing property lines adequately, and rendering them permanent, is best realized when one is confronted with the task of producing land surveys acceptable to the U. S. Department of Justice for the purpose of recording land ownership of sites for Federal Buildings and post-offices.

Experience in completing more than 1200 such plat surveys, scattered over the entire United States in cities, villages, and suburban locations, has convinced the writer that recorded descriptions are generally defective, and can be harmonized with existing physical evidence on the ground only by a process of compromise between all parties interested.

It is generally agreed that a parcel of land is definitely located on the ground surface, if and when all the boundary corners are monumented, and after such monuments have been placed with the mutual consent of all interested parties (which include the owner, his adjacent neighbors, and the municipality).

A survey, showing the lengths of all monumented property lines and the measured internal angles at all corners, with dimensions of the corners to the

[8] Cons. Engr., Harlingen, Tex.

[8a] Received by the Secretary April 6, 1939.

established street lines (or existing street lines when none are established), constitutes a complete record for all legal purposes. Such a survey constitutes the basis for writing a valid description of property to be acquired for public buildings. Thenceforth, the corner monuments will constitute the best evidence of lot lines and ownership as against any other claims, or descriptions, to the contrary.

The matter of supplying further references to a system of rectangular co-ordinates, or geodetic positions, established by other Government agencies, would not solve any location problems, nor add materially to the permanency of property lines previously fixed in position by permanent corner monuments. In an intensive public buildings program, with congressional appropriations contingent on prompt action, any delay in the acquisition of building sites might easily nullify the undertaking. The procedure requiring references to two-grid monuments, $\frac{1}{2}$ to 1 mile distant, as proposed by the Committee, would add only another obstacle to Government business which is already sadly encumbered with unnecessary technicalities.

Streets, public alleys, and highways occupy certain exclusive portions of land, the boundaries of which must be fixed definitely prior to locating other property lines. This can be accomplished best by monumenting the axial lines of thoroughfares, thus fixing, permanently, the property lines on all street frontages. The challenge to the surveyor is usually to find the street lines, in the absence of monuments, so that he may locate the property frontage and dividing lot lines. Few eastern cities have street intersection monuments, whereas the more progressive western municip lities are generally monumented.

With the intersections of street axes fixed by monuments, and the widths of streets taken from public records, the frontage property lines of lots are readily located. In the case of corner lots, the two adjacent street lines, and the included angle, are thus determined, and for an inside lot the distance to the nearest street corner measured along the lot frontage should appear on the survey to mark the point of beginning for the description of the lot in question.

The proposal of instituting a grid system of co-ordinates in each State has merit within certain limits, but cannot be accepted as a cure-all for property-line survey problems as claimed. If, and when, street monuments have been provided, and tied into the grid system, that should answer all practical needs. To go beyond this and give co-ordinates for all lot corners would be wasted effort. Furthermore, if a description is written in terms of property-corner co-ordinates, it would require considerable computation to ascertain the lengths of sides and internal angles of the plat, which are the important dimensional data.

Unless monuments of the grid system are available in close proximity to the property to be surveyed, their value is questionable. The best location for such monuments is undoubtedly at the intersections of street axial lines; and every city street and highway should be so monumented.

The best way known to safeguard property and other monuments is by State and municipal laws inflicting severe penalties on those found disturbing

or destroying such monuments. With the permanency of property monuments assured, the way will be cleared for the surveyor to function as he should.

The use of bearings, whether true or magnetic, leads to many complications without contributing anything of value either for purposes of locating or of describing property lines. It is an old custom still adhered to by some, but should now be regarded as obsolete. Bearings in a grid system, of course, are subject to variations due to the convergence of meridians.

Monuments suitable for marking public or private property consist of concrete posts 6 in. by 6 in. by 3 ft, set in the true corners and flush with the ground or grade level. For the intersection of street center lines a similar sub-surface concrete monument should be placed with a bronze plate set into the pavement surface. Although there is a remote possibility that a property monument may be molested or destroyed, it would be a simple matter to replace it without making use of co-ordinates. The trouble is that, outside of Government property, the practice of using monuments has not been generally adopted, and hence there are no monuments to become lost.

The Committee would do well to stress this matter as one of the greatest importance, and to add a paragraph to the Model Act stipulating a penalty for molesting, disturbing, or destroying any monument, bench-mark or triangulation marker placed by surveyors of the United States, State, county, or city where located for purposes of marking property lines, elevations, or points of the grid system, except by a duly registered surveyor, acting with authority.

The enormity of the undertaking to establish grid monuments in all the States will probably provide work for several thousand surveyors for many years to come. Much of this will devolve on the U. S. Coast and Geodetic Survey, whereas the local monumenting of street and highway axes will likely be done under the supervision of city engineers and State Highway Departments.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EARTHQUAKES AND STRUCTURES

Discussion

BY WALTER L. HUBER, M. AM. SOC. C. E.

WALTER L. HUBER,[26] M. AM. SOC. C. E. (by letter).[26a]—A basic assumption not mentioned in the authors' development of the equations for the vibration of an elastic cantilever is that the formulas hold only for the "steady state" condition—that is, the condition obtained only after a considerable interval of time has elapsed since the initial harmonic impulse. This one assumption alone, not including those pointed out by the authors, is difficult to substantiate in such an erratic ground movement as experienced during an earthquake. At the same time, since this "steady state" condition is not likely to be realized in an earthquake, it suggests the application of the impact theory as developed by Prof. Joseph N. Le Conte in 1926.[27] During earthquakes there is considerable evidence of failure due to impact. In reviewing the damage to the concrete stack of the Seal Beach Station of the Los Angeles Gas and Electric Corporation, due to the Long Beach earthquake of March 10, 1933, the application of Professor Le Conte's paper was studied as one line of analysis.

This stack, weighing about 2 800 tons, was 317 ft high, rising from a concrete platform which was, in turn, supported just beneath the roof on a steel framework. The base of the stack was about 65 ft above the ground. It was 24 ft in diameter at the top and 35 ft 6 in. in diameter at the base. The principal reinforcement consisted of vertical steel bars. Circumferentially only two layers of steel fabric were supplied for reinforcement. The stack suffered considerable damage in the earthquake, particularly in the middle third of its height, where many horizontal cracks opened. In certain places it appeared that such cracks had opened and had then come together. This was particularly noticeable where reinforcing steel appeared to have been first elongated and then compressed.

NOTE.—This paper by Leander M. Hoskins, Esq., and John D. Galloway, M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. Homer M. Hadley, and R. McC. Beanfield; and April, 1939, by Messrs. R. S. Chew, Jacob J. Creskoff, and Arthur C. Ruge.

[26] Cons. Engr., San Francisco, Calif.

[26a] Received by the Secretary April 10, 1939.

[27] "The Stresses in a Free Prismatic Rod Under a Single Force Normal to Its Axis," by J. N. Le Conte. *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 968.

Determination of forces due to earthquakes to which complicated structures are subjected is not an easy task, and opinions advanced from such determinations are not always beyond question. On the roof of Seal Beach Station three 20 000-gal water-tanks would seem to afford opportunities for rather direct determinations. Even in this case, however, certain very essential information was not available. For instance, it was not known whether, at the time of the earthquake, the tanks were filled. It seems probable that two of them were not filled, but that the third one may have been, and probably was, approximately full. Assuming that it was filled and assuming the ultimate strength of certain rods in the tank frame which were broken by the earthquake to be 65 000 lb per sq in., the force necessary to have caused the destruction of these rods would have amounted to 27% of the force of gravity; or, if the ultimate strength of the rods were assumed equal to 70 000 lb per sq in., this force would have been 30% of the force of gravity. Of course, if the tank were not filled, the force exerted was a larger percentage of the force of gravity since in any case the rods were broken, and if the tank were not filled they would have been acting upon a smaller mass. These percentages must be considered the minimum force. It is not known how much this force exceeded that necessary to rupture the rods.

The tank in question was supported on the roof of the station about 65 ft above the ground. On this roof it is probable that both accelerations and displacements were considerably more than those of the ground beneath the station. Since the earthquake of June 10, 1933, instruments have been installed by the U. S. Coast and Geodetic Survey to record accelerations in the top stories and in the basements of several buildings in California. These instruments, although of the strong-motion type, are so connected that a movement sufficient to start one instrument will start both, thus giving simultaneous records. These instruments have shown (as in the case of a minor earthquake of June 25, 1933, for example) considerably more acceleration in the upper parts of the buildings than on the ground. There may be cases where the acceleration of the ground will be more than that of the roof. The comparative accelerations depend upon the relation of the period of the earthquake to that of the building and many other somewhat indefinite conditions. All evidence indicated that for the earthquake of March 10, 1933, the acceleration at the roof of the Seal Beach Station (the base of the tank) considerably exceeded the acceleration at the ground. The base of the tank was at about the same elevation as the base of the stack, and a substantial concrete roof slab which remained unbroken in this region of the building afforded assurance that the motion of these two elements of the building was not very different. It is entirely reasonable, from all of the evidence available, to assume that the base of the stack was subjected to an acceleration of at least 25% of that of gravity. How much more than this percentage is a question. It may have been substantially more.

The stack was designed to withstand a lateral force of 20% of that due to gravity, applied at the center of gravity of that part of the structure above any section. This is the ordinary "cantilever method"; and, although it is applicable if static loads or fundamental vibration forces are to be resisted,

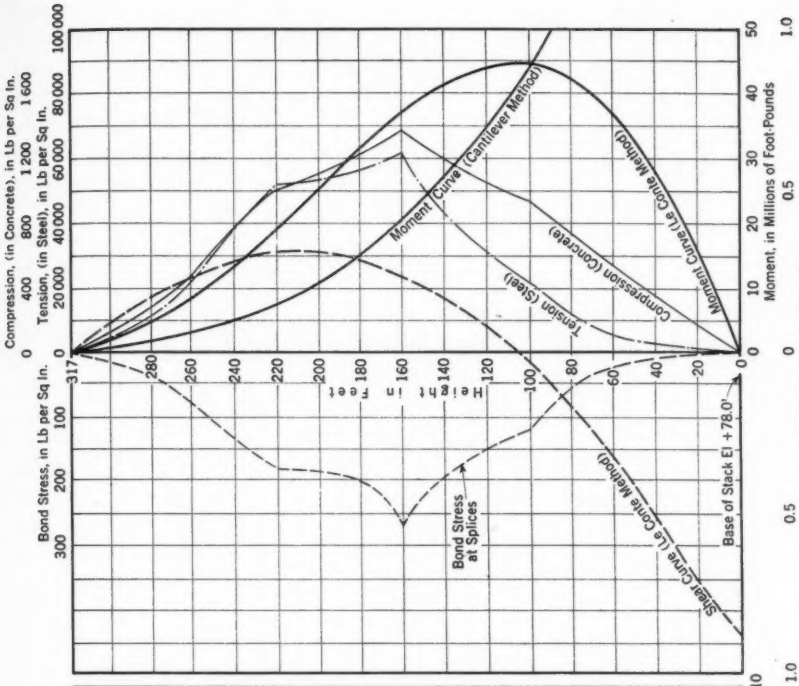
Compression, (in Concrete), in Lb per Sq In.
Tension, (in Steel), in Lb per Sq In.
Bond Stress, in Lb per Sq In.
Height in Feet
Moment Curve (Cantilever Method)
Moment Curve (Le Conte Method)
Compression (Concrete)
Tension (Steel)
Bond Stress at Splices
Shear Curve (Le Conte Method)
Base of Stack El +78.0'
Moment, in Millions of Foot-Pounds
Shear, in Millions of Pounds

FIG. 7.—SEAL BEACH STACK; STRESSES BY LE CONTE METHOD

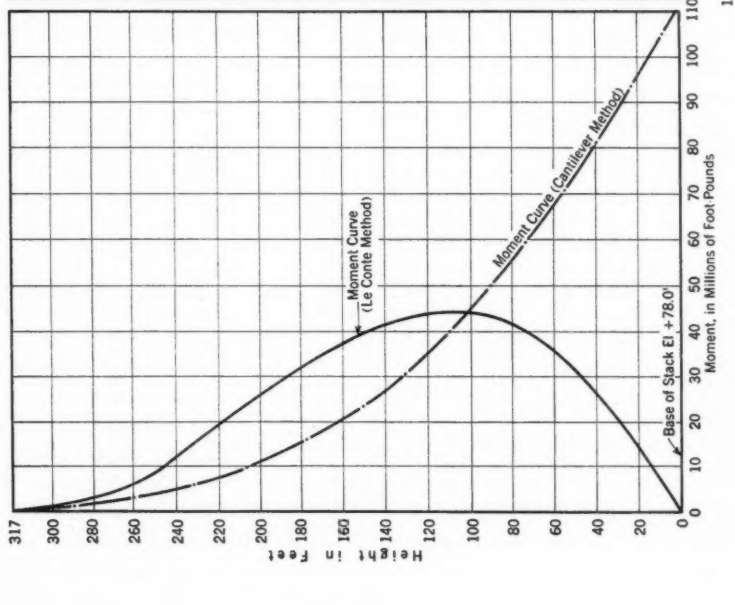
Height in Feet
Moment Curve (Le Conte Method)
Moment Curve (Cantilever Method)
Base of Stack El +78.0'
Moment, in Millions of Foot-Pounds

FIG. 6.—SEAL BEACH STACK; COMPARISON OF MOMENTS BY CANTILEVER AND LE CONTE METHODS

it is not in accordance with methods now commonly accepted for earthquake analysis of tall slender structures. Nor does it accord with observed destruction of such tall slender structures. Analysis in accordance with the "cantilever method" indicated that, for a force of 20% of gravity, the reinforcing steel would not be stressed beyond 16 000 lb per sq in. in tension, and the concrete, beyond 650 lb per sq in. in compression, which are conservative working stresses.

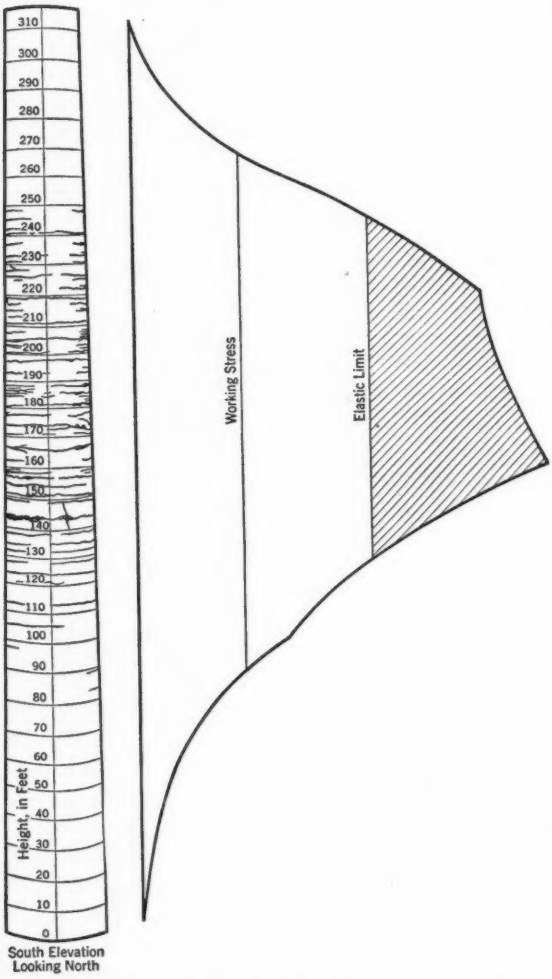

FIG. 8.—RELATION OF TENSILE STRESSES IN STEEL TO DAMAGE OF STACK

Professor Le Conte's method of analysis requires some assumptions which are not realized in their entirety, but which were particularly adaptable to this stack. These assumptions are that the structure shall be rigid, prismatic, and free of all support. Actually the stack was elastic, not truly prismatic, and had an elastic support, but these conditions approximated the assumptions required to be made.

A comparison of the moments derived by the cantilever method with those derived by following the method outlined by Professor Le Conte is shown in Fig. 6. The moments shown there are derived by applying a force of 25% of that of gravity. It will be noted that for that part of the stack near the base, the cantilever method gives moments far in excess of those of the Le Conte method, and it is also of interest to note that the lower part of the stack was little damaged. On the other hand, above one-third of the height the moments are found to be more than those for which the stack was designed—in places more than double. It is also interesting to note that at the very points where this analysis indicates largest moments and over-stressing of the steel, the stack suffered considerable damage.

Fig. 7 shows the stresses in the steel and concrete corresponding to the moments and shears obtained by the Le Conte method of analysis. For heights of from 150 to 225 ft, or from 47% to 71% of the total height, the tensile stress in the steel reinforcement was found to be more than 50 000 lb per sq in. and more than 60 000 lb per sq in. at a height of 160 ft, these stresses being beyond the elastic limit and approaching the ultimate strength of the material. It should be noted that unit stresses of steel are a function of the amount of steel actually supplied as well as the moment to be resisted—hence, the irregular curve. The computed bond stress ranged to as much as 266 lb per sq in., and it is not improbable that failure from this weakness resulted.

In Fig. 8 the very close agreement between computed stresses and the experienced cracking and failure of the stack is clearly shown.

Here is a practical example where the necessity of satisfying more than merely the laws of statics is clearly necessary to produce an earthquake-resistant structure. In applying mathematical analyses to this problem the writer was guided by detailed studies of damage by the earthquake, which is a test to be highly regarded. Every such mathematical analysis is only as valid as the assumptions on which it is based. Too often assumptions are made without a firm foundation, and thereafter the analyst is inclined to forget the doubts which troubled him when making the assumptions and to accept the results of the intricate mathematical analysis as the final solution of the problem. Although the stresses found in this investigation were, in all cases, considered to be approximate, the application of the theoretical analysis is substantiated by damage observed in careful and comprehensive inspections of the structure.

The theoretical analysis discussed herein did not enter into the problem of reconstruction because it was found necessary to remove a considerable part of the stack without replacing it. The problem was thus removed from the field of this analysis.

The experience noted was in accord with that of other stacks built on the tops of buildings and which have been subjected to earthquake shocks. Homer M. Hadley, Assoc. M. Am. Soc. C. E., in commenting upon damage resulting from the Japanese earthquake of September 1, 1923, states:[3]

> "Of all structures of reinforced concrete, chimneys gave the least satisfactory performance. When built up from the tops of buildings, they failed in all cases."

An attempt was made to apply the so-called "hammer theory" in an analysis of the stack, utilizing some of the work of Walter Ruppel, Assoc. M. Am. Soc. C. E.[3] By this theory moments somewhat larger than those heretofore enumerated in this discussion were found in the lower 90 ft of the stack, but these were all less than the moments for which this part of the stack was designed. Above this lower section, although following the same general shape, the moment curve showed somewhat smaller moments. This condition does not accord with observations of damage, although even in this case the

[3] Unpublished Report of the Special Committee of the American Society of Civil Engineers on Effects of Earthquakes on Engineering Structures with Special Reference to the Japanese Earthquake of September 1, 1923. Available for reference at Engineering Societies Library, 33 West 39th Street, New York, N. Y.

vertical reinforcement was stressed beyond its elastic limit. It is the writer's opinion that the base of the particular stack under consideration did not have enough effect to make the "hammer theory" applicable.

It would be interesting to investigate the stack by means of the vibration theory developed by the authors, but this is not possible with available data. There is some difficulty in determining the period of ground motion from available seismograph records. These records verify the authors' statement (see Part I "Earthquakes and Their Effects") that:

> "The waves that damage structures are impressed waves, changing rapidly in period, amplitude and acceleration. Seismograms rarely show traces of harmonic waves, or those with regular characteristics. This is probably a very fortunate circumstance because it prevents, to a large extent, the resonance of vibrating structures."

In the present case, there is even less information to indicate the period of motion at the roof of the power-house, or the base of the stack.

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DISCUSSIONS

BEACH EROSION STUDIES

Discussion

BY MESSRS. GEORGE A. SOPER, AND JAMES J. O'ROURKE

GEORGE A. SOPER,[5] M. AM. SOC. C. E. (by letter).[5a]—Engineers will be indebted to the author for setting forth with such admirable clearness the resources which have been developed by the Beach Erosion Board for the investigation of beach erosion problems along the sandy coasts of the United States, and for the recommendations he has made for the construction of bulkheads and groins.

The point of view of the engineer who is called upon to protect a piece of property is not always one which is favorable to exhaustive investigation. He must be satisfied with what is necessary and sufficient; for the engineer has a specific, and usually a limited and pressing, problem that he must solve, and he must solve it at the least expenditure of time and money. His investigations must be restricted to what is absolutely required. He cannot carry on elaborate studies, some of which have comparatively little bearing on the particular problem in hand, but tend to build up the science and art of beach protection.

The writer strongly indorses what the author says about the value of inquiries into the history of the particular locality in question, with reference to destructive storms and the fate of structures that have been built at this place. Of the highest importance, also, is the necessity of assembling at the earliest moment such evidence as already exists concerning the probable effects of changes, natural and otherwise, that have been made in the vicinity of the place where the protective works are to be located. As early as possible a rapid survey should be made to see what information may be available and how much reliance may be placed upon it. It will sometimes be surprising to find what useful leads personal testimony may supply. Often this is all the evidence procurable. The opening of the Moriches Inlet and, in fact, the curious behavior of the entire beach at Fire Island, Long Island, N. Y., since

NOTE.—This paper by Earl I. Brown, M. Am. Soc. C. E., was presented at the meeting of the Waterways Division at Jacksonville, Fla., on April 21, 1938, and published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Morris N. Lipp, M. Am. Soc. C. E.

[5] Cons. Engr., New York, N. Y.

[5a] Received by the Secretary April 12, 1939.

the earliest times, have much light thrown upon them when such information is taken into consideration.

Human testimony sometimes affords the only clews which the engineer can obtain concerning storms; and, it should be emphasized that it is storms, and not average weather conditions, in which he should be particularly interested. Unfortunately, lists of storms are not kept by the U. S. Weather Bureau or any other branch of the Government. Nor is the severity or damage done by them often made the subject of an engineering report.

An investigation of the subject of destructive storms along the Atlantic Coast—their frequency, duration, damage done, and the effectiveness of the measures taken to protect property against them, with the practical lessons to be derived therefrom—would be a valuable contribution for the Federal Government to make.

The writer is in favor of inducing the U. S. Coast Guard to make more definite and accurate records of the weather, and he would have observations of high and low tide, the angle of approach of the waves, the height of the tide, and the configuration of the beach included in the log.

More information is needed concerning the general movements of the coast line than can always be had from the maps and surveys that are available. Is the south side of Long Island retreating, for example, and if so, to what extent? Is it subsiding and, if so, at what rate? To what extent does the formation of bays and points along the shore represent continuous tendencies, and how can this be determined when a system of bulkheads and groins, say, 1 000 ft long, is to be constructed?

The author has said little about the movement of sand under the influence of wind and in the regions above the reach of the ocean; and yet this would seem to be an important part of the beach erosion investigations. He has stated that sand traps and their proper use might give considerable helpful information; but, will he not suggest how they should be constructed and used? The quantity of sand moved by the wind toward, and away from, a beach is at times very great. The writer has seen, from a distance, the atmosphere for a height of more than 200 ft look as clouded by sand as from smoke where a fire was raging. More often the sand is swept in drifting currents along the surface of the beach. The writer knows a situation in which the movement of dry sand has considerable effect in determining the proper means of beach protection. The question of sand fixation is closely related to that of beach erosion and is by no means a simple matter.

The writer is in hearty accord with Colonel Brown as to the need of placing as long a stretch of beach as practicable under the protection of a single comprehensive scheme, but he does not see how an engineer who is called on to build a structure in front of a house or along any other limited water front can control this very important matter. No matter how well his work is done, if the scope is not adequate, the result will be more costly and less effective than it should be.

Among the elements of design mentioned by the author one which seems especially worthy of endorsement is the need of keeping the water from getting behind a bulkhead; this is imperative, whether it gets there over the top or

around the ends. It is almost impossible to get such water out, once it gets in; but if the author has some simple way of accomplishing it, it is hoped he will describe it in his closing discussion. It seems much better to keep the water from getting behind the bulkhead in the first place. It may be prevented from getting around the ends by bringing them back to high land or to another bulkhead, if there is a good one at hand; but against water coming over the top the best plan seems to be to back up the bulkhead to the top with sand or other solid filling, and keep it filled. The author seems to think this is not a very expensive procedure, but the writer believes it is generally so. Suction dredging is, by no means, always available. It is not practicable on the open seashore unless there is a plentiful source of supply in the rear, and any other method of handling sand is expensive.

JAMES J. O'ROURKE,[6] ESQ. (by letter).[6a]—Under the heading "Observations on Design of Protective Works" Colonel Brown suggests a "bulkhead and groin system" or a "groin" system. These will be successful where there is a considerable quantity of sand carried by the littoral drift. Otherwise, such groins are unsuccessful and Colonel Brown recommends that an artificial supply of sand be pumped between the groins, but admits that this supply may have to be renewed from time to time. This requires considerable money for maintenance.

The Outer Drive in Chicago, Ill., through Lincoln Park, extends parallel to Lake Michigan from North Avenue to Fullerton Avenue. The east curb (that is, the one nearest the Lake) is separated from it by a paved concrete beach about 60 ft wide. The elevation of the drive is about + 10. The paved beach slopes from the roadway at Elevation + 10 to the old sea-wall at Elevation + 2. The elevation of Lake Michigan since 1919 has averaged about 1 ft below datum. In 1929 there was an unprecedented rise in the waters of Lake Michigan to an average level of above + 2 during the summer months, which is the approximate elevation of the top of the old sea-wall. This rise, accompanied by strong winds off the Lake, destroyed part of the drive.

In 1930 a series of groins each about 500 ft long and spaced 600 ft apart were built along this stretch of roadway and shore line, inclined at an angle of about 70° from the north, the idea being that a considerable quantity of sand would be accumulated between the groins and thus protect the roadway from future storms. These jetties did not accumulate any sand in the 9-yr period since that time because there is practically no littoral drift at this point. Fortunately, however, there has been no high water since 1929 and no further damage has resulted on account of storms.

In 1939 the Commissioners of the Chicago Park District, who control the shore line of Lake Michigan at this point, decided to build a bathing beach with the two-fold purpose of protecting the roadway from future storms and creating much needed bathing facilities for the people of Chicago. In 1938, 1 800 ft of bathing beach had been completed.

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[6] General Eng. Designer, Chicago Park District, Chicago, Ill.

[6a] Received by the Secretary April 7, 1939.

A partly submerged bulkhead (Fig. 12) was constructed, joining the lakeward ends of four adjoining jetties. This formed three bays, each 500 ft long and 600 ft wide, into which sand was pumped to an elevation of + 5 at the

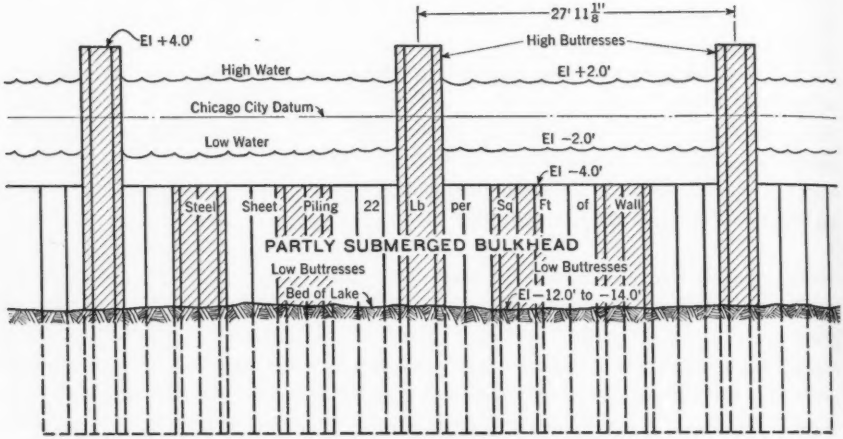

FIG. 12.—ELEVATION OF PARTLY SUBMERGED SEA-WALL

old sea-wall and + 8 at the new, partly submerged bulkhead. This created a sloping beach from a point about half-way up on the old paved beach to the new bulkhead, the old sea-wall being covered up in the operation and no longer serving any purpose.

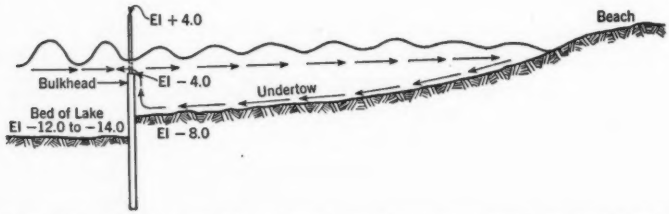

FIG. 13.—SECTION OF BEACH SHOWING STILLING ACTION OF UNDERTOW

This method of shore protection differs from that recommended by Colonel Brown in that the bulkhead is eliminated at the shore end of the groins and the partly submerged bulkhead substituted at the lakeward end. The advantages of the latter over the former are:

(1) It is much cheaper; the partly submerged bulkhead costs from one-third to one-quarter as much as a sea-wall along the shore; and,

(2) It effectively holds the sand behind it, whereas with the former the danger of sand being carried away is always present.

In operation, the partly submerged sea-wall breaks up the sea in this manner. The portion of the wall above the water partly impedes the waves. All the water that passes over it must return to sea in some way. It goes back

in the form of an undertow along the bottom. When this undertow reaches the submerged part of the sea-wall it cannot travel any farther along the bottom. In order to reach sea it must rise to the surface of the water and

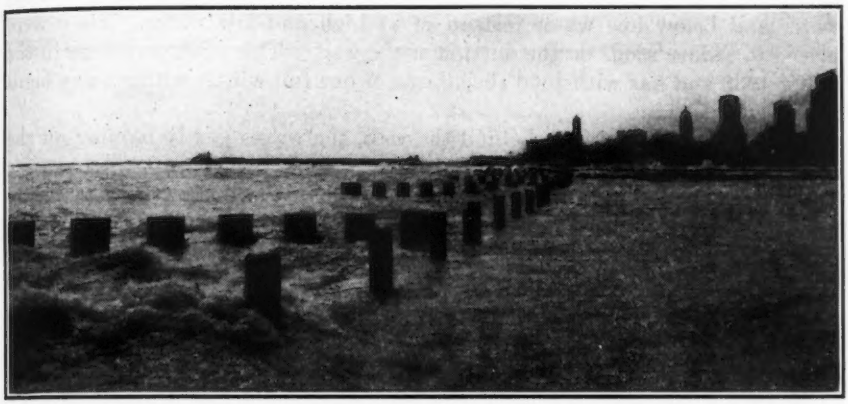

FIG. 14.—SHORE-LINE WAVES AT SEA-WALL; NEW NORTH AVENUE BEACH, LINCOLN PARK, CHICAGO, ILL.

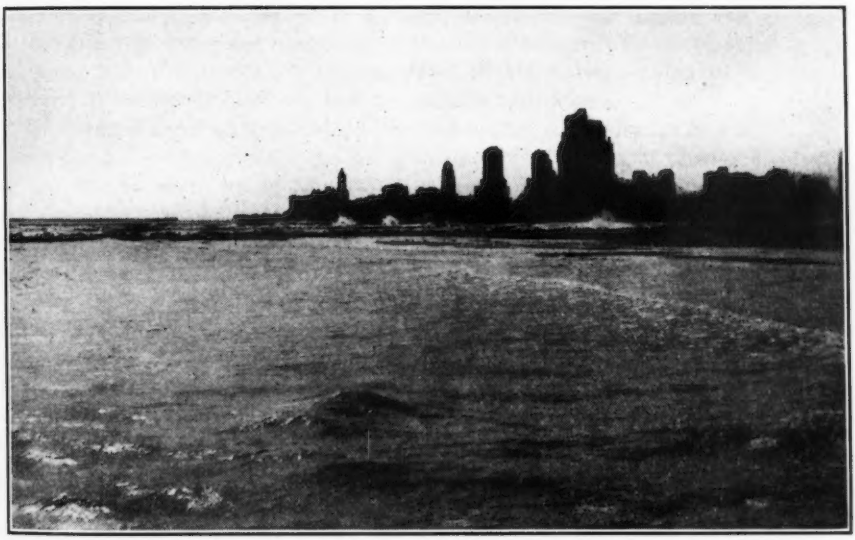

FIG. 15.—SHORE-LINE WAVES BEHIND SEA-WALL; NEW NORTH AVENUE BEACH, LINCOLN PARK, CHICAGO, ILL.

then, in trying to escape, it encounters the sea coming in, thus causing a stilling action at the wall. This is shown in diagram in Figs. 13 and 14.

Fig. 14 is a view photographed on March 27, 1939, with a wind of 30 miles per hr blowing from the northeast at about right angles to the shore. It was taken at low water, approximately Elevation − 1. Just to the left of the

partially submerged sea-wall are partially submerged jetties 75 ft long and spaced at intervals of 200 ft, somewhat similar to the ones referred to by Colonel Brown, which were constructed in 1936 at Palm Beach, Fla., with the exception that those in Fig. 14 have alternate sets of piles driven above high water and below low water instead of at high and low water. They were placed to reduce scour on the outside of the wall. This work was done in the fall of 1938 and has withstood the storms of one full winter without any signs of movement.

Fig. 15 shows the beach behind the wall, the waves gently lapping on the shore, their force dissipated by it. In contrast the waves from the open sea can be seen pounding against the old sea-wall in the distance. The partly submerged sea-wall has the high buttresses spaced at intervals of 27 ft $11\frac{1}{2}$ in. In future installations this interval will be reduced to 14 ft to assist in breaking the waves at high water.

This work was done under the general direction of George T. Donoghue, M. Am. Soc. C. E.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN OF CIRCULAR CONCRETE TANKS

Discussion

BY MESSRS. CARL A. ROSENGARTEN, AND I. K. SILVERMAN

CARL A. ROSENGARTEN,[6] ASSOC. M. AM. SOC. C. E. (by letter).[6a]—Although a circular concrete tank is economical so far as having the least perimeter for a given area, it would obviously be extravagant if it were designed only as a common cylinder with all stresses taken by the steel, and if a perfectly good structural material, such as concrete, were used merely as a filler. A design that utilizes the structural properties of the concrete more efficiently, therefore, should be more economical, as demonstrated in Mr. Salter's paper in which he considers the lower part of the wall to act as a cantilever.

However, if the wall were considered as bending vertically for its full height from a fixed-end condition at the base, to a supporting ring of tension steel at the top, the concrete would be utilized to a still greater extent as more of the wall would be stressed in bending. Water-tightness can be improved by prestressing the steel tension ring at the top which, again, is made possible by increasing the use of the concrete in compression.

In designing concrete tanks of various shapes, the writer has found that, wherever it is possible to secure a support at the top of the wall, it is generally more economical to design the wall spanning vertically. This is usually true because the triangular loading of liquid pressure is peculiarly adapted to this condition. The greater part of the load (two-thirds) goes to the bottom of the wall where the floor is already available as a support, leaving only one-third of the load to be taken care of by beams or ties at the top. A fixed-end condition at the base provides a still more favorable distribution of load, giving a reaction of four-fifths of the load at the bottom and one-fifth at the top.

Without attempting to check the rigorous mathematical analysis, the writer notes, in Mr. Salter's illustrative example, that he has determined the area of ring steel by dividing ring tension by a constant unit stress of 12 000 lb per sq in. The stretch or elongation of the steel in the tension rings varies

NOTE.—This paper by George S. Salter, M. Am. Soc. C. E., was published in March, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Architectural Engr., The Ballinger Co., Philadelphia, Pa.

[6a] Received by the Secretary March 30, 1939.

from a maximum near the top of the wall to a theoretical zero at the bottom, according to the deflection of the cantilever wall. According to the fundamental theory that the stress is proportional to the strain or elongation, the unit stress in the ring steel must also vary from a maximum for the rings near the top of the wall to zero for a ring at the bottom.

Since the distribution of stress in any two-way system is complicated by many variables, it would seem advantageous to simplify designs wherever possible without sacrificing economy. In the case of relatively low tanks this can be accomplished by considering the wall as a one-way slab, spanning vertically from a fixed-end condition at the base to a support at the top consisting of a pure tension ring. If this ring is not pre-stressed, adjustment to the vertical steel can be made for the deflection (or, settlement of support) caused by the elongation of the top ring steel.

Corrections for *Transactions*: In March, 1939 *Proceedings*, Page 421, Line 4, change "Fig. 1 (*a*)" to "Fig. 1 (*b*)"; ordinates to Figs. 2 and 3 are "values of θ"; on Page 425, Line 4, change "$1.00\,\pi$" to "$0.40\,\pi$"; and, on Page 426, Line 13, change "C_m" to "C_t."

I. K. SILVERMAN,[7] Assoc. M. Am. Soc. C. E. (by letter).[7a]—It is interesting to note that the problem discussed by Mr. Salter is a special case of a more general problem which includes the arch dam. The solution of the problem of the cylindrical tank has been known and used for many years, its first solution being given by H. Müller-Breslau.[8]

The author has neglected the Poisson effect which is equivalent to assuming that m (Poisson's number) is equal to infinity. Equation (10*b*), however, can be obtained from Müller-Breslau's solution by inserting $\mu = \frac{1}{m} = 0$. The type of formula exemplified by Equation (10*b*) appears throughout engineering literature in the discussion of problems of beams on elastic foundations, and therefore puts the problem of the circular tank in that category. Thus, the cantilever elements carry the difference between the water load and the reaction furnished by the ring elements. This reaction is proportional to the value of the deflection y. This statement is expressed by Equation (10*b*). The arch dam subjected to water load is amenable to the same method of attack, and the trial-load method for the analysis of arch dams is based on the equating of the deflections of the horizontal (arch) elements to those of the vertical (cantilever) elements. The loads on the cantilever elements are similar to that shown in Fig. 1.

Due to symmetry, the forces acting on an element in a cylindrical tank of varying wall thickness are as follows:

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[7] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[7a] Received by the Secretary March 31, 1939.

[8] "Berechnung von Behältern," by Th. Pöschl, Berlin, 1926, Second Edition, p. 132 ff; "Beton und Eisen," Vol. 7, 1908, p. 150; "Concrete and Constructional Engineering," by H. Carpenter, April, 1927, and June, 1929; "Reinforced Concrete Reservoirs and Tanks," by W. S. Gray, London, pp. 11–15; and "Arched Dams," by B. A. Smith, *Transactions*, Am. Soc. C. E., Vol. 83 (1919–1920), p. 2027.

Equilibrium against moment:

$$Q = -\frac{dM}{dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

equilibrium in the radial direction:

$$-\frac{dQ}{dz} + \frac{T}{r} = w\,z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

and, equilibrium in the vertical direction:

$$\frac{dS}{dz} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

The movement in the radial direction is denoted by y. The corresponding movement in the vertical direction is given by "u." The strains in the radial and vertical directions are accordingly defined as:

Radial—

$$\epsilon_1 = \frac{y}{r} \quad \ldots\ldots\ldots (21a)$$

and, vertical

$$\epsilon = \frac{du}{dz} \quad \ldots\ldots\ldots (21b)$$

From the generalized Hooke's law the stress on the element in the vertical direction is

$$\frac{S}{t} = \frac{E}{(1-\mu^2)}(\epsilon + \mu\,\epsilon_1) \ldots (22a)$$

The tangential hoop stress is

$$\frac{T}{t} = \frac{E_0}{(1-\mu^2)}(\epsilon_1 + \mu\,\epsilon) \ldots (22b)$$

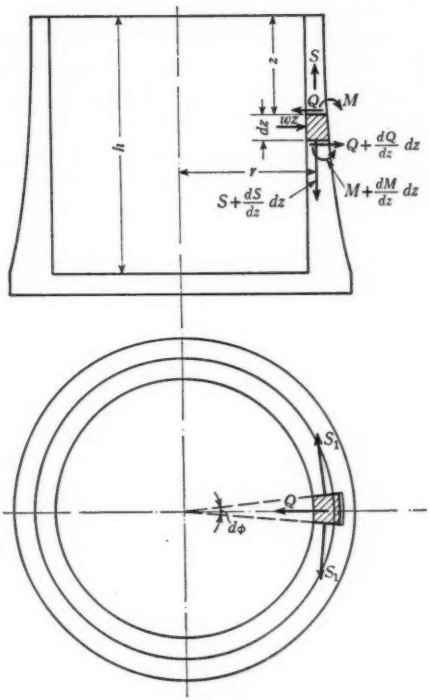

FIG. 6

in which μ = Poisson's ratio = $\frac{1}{4}$ to $\frac{1}{6}$ for concrete. From considerations of curvature

$$M = \frac{E\,I}{(1-\mu^2)}\frac{d^2y}{dz^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Integrating Equation (20c), using the free boundary condition at $z = 0$, together with Equations (21) and (22a), $S = 0$, or

$$\epsilon + \mu\,\epsilon_1 = \frac{du}{dz} + \mu\frac{y}{r} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Using Equations (20*a*), (20*b*), (23) and (24) the differential equation for the radial movement, y, is as follows:

$$\frac{d^2}{dz^2}\left(I\frac{d^2y}{dz^2}\right)+\frac{E_0\,t}{E\,r^2}(1-\mu^2)\,y=\frac{w\,z\,(1-\mu^2)}{E}\quad\ldots\ldots\ldots(25)$$

which reduces to Equation (10*b*) when I is constant and $\mu = 0$, $z = (h - x)$. Since $(\epsilon_1 + \mu\,\epsilon) = (1 - \mu^2)\,\epsilon_1 = (1 - \mu^2)\,\frac{y}{r}$ the hoop tension is

$$T=\frac{E_0\,t}{r}\,y\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

so that the unit tensile hoop stress is proportional to the radial movement y.

When the walls of the tank have a triangular section, Equation (25) can be integrated in terms of Bessel functions as shown by F. Tölke.[9] For a trapezoidal section or one bounded by a parabola and a straight line approximate methods are available.[10]

[9] "Der Bauingenieur," by F. Tölke, January 8, 1937.

[10] "Berechnung von Behältern," by Th. Pöschl, p. 155 ff.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LATERAL SPILLWAY CHANNELS

Discussion

BY MESSRS. W. E. HOWLAND, LEWIS V. CARPENTER AND JOHN K. VENNARD AND FENNER H. WHITLEY, F. KNAPP, AND CARL ROHWER

W. E. HOWLAND,[6] ASSOC. M. AM. SOC. C. E. (by letter).[6a]—The experimental data quoted in this paper clearly establish the validity of the basic differential equation used and the rational methods developed appear to the writer to be of great value in analyzing, completely, the behavior of lateral spillway channels. Because the integrated equations require adjustment by trial and error methods the writer has investigated the use of similar methods applied to the basic differential equation itself. Results indicate some value in this somewhat different method of approach.

Equation (17) may be rewritten as follows:

$$dy = \frac{1}{g}\left(\frac{f\,V^2\,dx}{8\,R} + 2\,V\,dV + \frac{dA}{A}\,V^2\right) \ldots\ldots\ldots\ldots (38)$$

since

$$q\,\frac{V^2}{Q} = \frac{dQ}{dx}\,\frac{V^2}{Q} = \frac{dQ}{dx}\,\frac{V^2}{A\,V} = \left(\frac{A\,dV + V\,dA}{dx}\right)\frac{V}{A} = \frac{V\,dV}{dx} + \frac{V^2}{dx}\,\frac{dA}{A} \ldots (39)$$

in which, in addition to the notation of the paper, A = area of cross-section of channel stream; otherwise symbols are those of the author. Equation (39) may be expressed as follows:

$$dy = 2\,d\left(\frac{V^2}{2\,g}\right) + 2\,\frac{V^2}{2\,g}\,\frac{dA}{A} + \frac{f}{4\,R}\,\frac{V^2}{2\,g}\,dx \ldots\ldots\ldots\ldots (40)$$

or, in words: The drop in the water surface between two sections a distance dx apart is twice the difference between the velocity heads of these sections, plus twice the product of the (average) velocity head in this length and the proportional change in area between the two sections, plus the allowance for friction head loss in this length between these sections.

NOTE.—This paper by Thomas R. Camp, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Associate Prof., San. Eng., School of Civ. Eng., Purdue Univ., Lafayette, Ind.

[6a] Received by the Secretary March 9, 1939.

An approximate method for calculating the difference in surface elevations along the channel might be to divide its entire length into a number of smaller lengths and then to apply Equation (40) to these separate lengths, treating the differentials that appear in the equation as actual assumed or calculated differences; thus:

$$\Delta y = 2\,\Delta\frac{V^2}{2\,g} + 2\,\frac{V_{av}^2}{2\,g}\frac{\Delta A}{A_{av}} + \frac{f}{4\,R}\frac{V_{av}^2}{2\,g}\Delta x \ldots\ldots\ldots\ldots (41)$$

in which V_{av} and A_{av} are average values of velocity and area. The burden of cutting and trying in applying Equation (41) progressively to a number of short sections of the channel, by the usual step method of calculation, has been found to be troublesome; nevertheless the equation has proved to be useful as will now be shown.

Calculations indicate that the first term to the right of the equation sign is of predominant importance in estimating the value of Δy (the vertical drop of water surface between sections). Thus, if the elevation of the water surface at the down-stream end of the channel is known, as well as the flows, the velocity head may be computed. Then the water-surface elevation at the upper end of the channel may be estimated approximately as two of these velocity heads higher than the down-stream water surface. This may be shown by a consideration of actual data in Table 2, obtained from the paper.

TABLE 2.—OPERATING DATA, TESTS ON LATERAL SPILLWAY CHANNELS
(Units Are in Feet)

Experiment	Depth of water in channel down stream, read from Figs. 4, 5, and 6	Distance from upper end	Values of H_0		Observed overall difference of water-surface elevation in terms of velocity head at lower end
			Observed	Approximately calculated*	
Detroit, Mich.:					
Run No. 1	1.04	37.5	1.00	1.00	1.995
Run No. 2	0.91	37.5	0.82	0.80	2.06
Cleveland; Division Avenue	2.83	47.5	4.44	4.64	1.78
M.I.T.; Experimental Channel	0.58	20.0	0.80	0.83	1.76

* Two velocity heads added to down-stream water-surface elevation.

(Data for the experiment on the Baldwin Filtration Plant were not included because the writer was unable to correlate the data given for q and Q in this case, as given in Table 1.)

The same principle may be used in the reverse direction; that is, assuming the elevation at the upper end of the section, the elevation at the lower end may be estimated in this manner:

$$\Delta y = H_0 + S\,x - d = 2\,\frac{V^2}{2\,g} = 2\,\frac{q^2\,x^2}{b^2\,d^2}\,2\,g \ldots\ldots\ldots\ldots (42)$$

Knowing H_0, S, x, q, b and g, one may solve Equation (42) for d. Using this formula to estimate the depth at a distance of 37.5 ft down stream from the upper end, in the Detroit plant tests, one obtains, in Run No. 1, $d = 1.04$ ft and in Run No. 2, $d = 0.87$ ft.

In the other experiments Equation (42) results in no real roots owing to the slight errors in the theory evident from Table 2; but a slight readjustment of the values of H_0 will yield equations with real roots.

These approximate calculations—based upon the assumption that the drop in the water surface between the up-stream end of the channel and any other point is twice the velocity head at that point—will probably be satisfactory for most practical purposes. They also serve as a basis for a more exact method for the analysis of the channels of this study. The writer proposes the following procedure for this purpose:

Laboratory Study to Obtain f.—In a laboratory study to obtain f, from measurements of the elevation of the water surface in the channel at a number of points (such as are indicated by Professor Camp) determine depths, areas and velocity heads at these sections and average velocity heads and substitute these and other dependent calculable values in Equation (41) and then compute f therefrom for each separate length of channel. No "cuts" and "trys" are needed in this procedure.

Exact Analysis.—In an exact analysis of a design, compute the water-surface elevations and depths at a number of sections along the channel by means of the approximation already considered. When the depth of channel is given at the down-stream section, first compute the elevation at the up-stream section as two velocity heads higher. (If the elevation at this section is assumed this step would not be necessary.) Then establish approximate depths at a number of sections below by means of Equation (42), assuming x and solving for d by trial or, if it is preferred, assuming d and solving for x. The latter procedure is more simple because it involves a quadratic instead of a cubic and does not require a cut and try solution. In Equation (42) all other quantities would be known, of course. Use this approximate water surface to compute the second and third terms on the right side of Equation (41) for the several lengths into which the channel has been divided (that is, the lengths between sections at which depths have been computed). Add, algebraically, both sets of "minor" terms; that is, obtain $\sum 2\,\frac{V_{av}^2}{2g}\frac{\Delta A}{A} + \sum \frac{f}{4R}\frac{V_{av}^2}{2g}\Delta x$ for the entire channel and add this to the original estimate of Δy $\left(\text{that is, to } 2\,\frac{V^2}{2g}\right)$ to obtain the more exact difference between water surfaces at the extremities of the channel. Thus H_0 may be recomputed. If desired, all the intermediate elevations may then be recalculated from the following modification of Equation (41):

$$H_0 + s\,x - d - \sum_0^x 2\,\frac{V_{av}^2}{2g}\frac{\Delta A}{A} - \sum_0^x \frac{f}{4R}\frac{V_{av}^2}{2g}\Delta x = \frac{2\,q^2\,x^2}{b^2\,d^2\,2g} \quad \ldots (43)$$

The entire process could be repeated to obtain still greater precision starting with the newly determined water depths from Equation (43); but, since it appears (from Table 2) that very close approximations to the water surface were obtained with the first crude or simple approximation there is good reason to suppose that the foregoing procedure would yield satisfactory precision without repetition.

It is interesting to consider why the simple approximation (which neglects the second and third terms in Equation (41)) yields results which agree so well with measurements and with the results of more exact and elaborate methods of analysis. Apparently the reason is that these terms are of small value and tend to compensate each other. As given in Equation (41), the second term on the right side is predominately negative and the third is always positive. The second term would disappear absolutely if the bottom of the channel were parallel to the water surface, a condition which is approximated with inclined channels. This term becomes of greater importance as the channel is made more level, in which case it may more than compensate for the effect of friction (see the last two tests in Table 2). From the data given the friction term, itself, appears to be of very small importance, amounting to about 8% of the first term, in the first experiment quoted.

This study calls attention to the very considerable loss of energy which occurs in these channels due, not to friction, but to the impact of the slow moving, and fast moving, water. It suggests that an increase in capacity might result from the use of devices to deflect the incoming water into the direction of the main stream before it enters the stream. The writer would like to see tried the simple device of placing vanes, curved only in one direction (see Fig. 7). Such vanes might extend from the crests of the weirs down to the anticipated elevation of the water surface in the channel. It appears to be possible, theoretically, to increase the capacity of a channel as much as 50% by such energy-saving devices. Although the use of such devices might not be economical in an original design, it might be helpful in increasing the capacity of old channels that have proved to be somewhat inadequate.

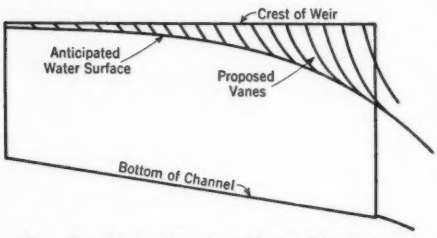

FIG. 7.—LONGITUDINAL CROSS-SECTION OF LATERAL SPILLWAY CHANNEL

To summarize: This study clearly establishes the validity of the basic differential equation employed. The writer suggests the use of the approximation—change in water-surface elevation in lateral spillway channels equals twice the difference in velocity heads between corresponding sections. This approximation may be used to advantage both for preliminary designs and also as the basis for exact analyses. Furthermore, the study emphasizes the great loss of energy that occurs from impact in these channels and suggests the possibility of improvements in design, to increase capacity.

LEWIS V. CARPENTER,[7] M. AM. SOC. C. E., JOHN K. VENNARD,[8] AND FENNER H. WHITLEY,[9] JUNIORS, AM. SOC. C. E. (by letter).[9a]—The problem treated in this paper has long been a vexing one for designers of certain types

[7] Prof., San. Eng., Coll. of Eng., New York Univ., New York, N. Y.
[8] Instr., Fluid Mechanics, Coll. of Eng., New York Univ., New York, N. Y.
[9] Instr., San. Eng., Coll. of Eng., New York Univ., New York, N. Y.
[9a] Received by the Secretary April 6, 1939.

of sanitary engineering structures. The author is to be commended for the completeness of his solution. He has taken the well-known Hinds[2] analysis and adapted the equations so that they are available for the design of such appurtenances as wash-water gutters and trickling filter collection troughs.

The Weisbach-Darcy friction factor that Professor Camp uses in his mathematical analysis of flow is well known. As this friction factor is used in the equations of the paper, it is made to embody all of the various resistance phenomena that occur throughout the length of the channel. Resistance to flow in a lateral spillway channel comes from two fundamentally different sources: (1) That of skin friction at sides and bottom of the channel (which alone usually determines the size of f); and (2) that resulting from the interference of liquid entering the channel normal to its axis. Undoubtedly the latter effect is the predominant one in determining the size of the friction factor in lateral spillway channels. This fact is borne out by the trend in friction factors shown in Table 1 in which a channel having inflow over the entire length of only one side (M.I.T. tests) and having thus a minimum interference has the lowest friction factor (0.033). Where inflow is distributed over the entire length of the channel but enters from both sides (Detroit tests) a greater interference is produced and a greater friction factor (0.055) results. Where there is a concentrated inflow from lateral gutters (Cleveland tests) a maximum interference occurs and results in a still higher friction factor (0.10–0.11). Although the friction factor data in Table 1[9b] are meager, they indicate the effects of various types of inflow upon this factor, and thus provide a guide for the designer in the assumption of a friction factor for a given design. It would be helpful in the selection of values of f if the author could present more data showing the variation of the friction factor with a variation in the rate of inflow into the channel.

Professor Camp's assumption (see heading "Critical Depth"), that "For most of the lateral spillway channels used in water and sewage plants, the depth at the lower end of the channel will be fixed at upper stage by the hydraulic conditions in the conduit down stream, or the flume will discharge freely," is correct. The further assumption that in the design of freely discharging flumes, a hydrostatic control section exists at a distance up stream from the end, equal to three or four times the critical depth, needs some qualifying statements.

Assume that in a rapid sand filter the lateral wash-water gutters are spaced 6 ft, center to center, and that they discharge freely. A vertical rise of 36 in. per min is used when washing the filter. The increment q will be 0.3 cu ft per sec per ft length of gutter, with one-half of the water coming in on each side. If the wash-water gutter is rectangular, the critical depth is given by Equation (28). In Table 3 the location of the hydrostatic control is calculated for lengths of 10, 20, and 40 ft and a breadth of gutter equal to 12, 15, 18, and 24 in., using the author's assumption. In the gutters at the Cleveland plant where the length is 49 ft, the author's assumption is fairly close because, at the hydrostatic control section, all of the water has entered the gutter.

[2] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 885.

[9b] Correction for *Transactions:* In Table 1, the value of q corresponding to the line "Fig. 5(b)" should read "1.5"

However, if the more typical lateral wash-water gutter is considered, in which the water is coming in both sides in equal increments, for a 40-ft channel, 2 ft wide, between 7.8% and 10.4% of the wash water has not entered the

TABLE 3.—EFFECT OF ASSUMING THAT THE HYDROSTATIC CONTROL SECTION IS UP STREAM FROM OUTLET AT A DISTANCE 3 TO 4 TIMES THE CRITICAL DEPTH

Length, l, in feet	Flow, Q, in cubic feet per second	Width, b, in inches	Depth, d_c, in feet	$3 \times d_c$, in feet	$4 \times d_c$, in feet	Percentage of Total Q Not in Channel	
						Hydrostatic control up stream $3 \times d_c$	$4 \times d_c$
10	3.0	12	0.65	1.95	2.60	19.5	26.0
		15	0.56	1.68	2.24	16.8	22.4
		18	0.50	1.50	2.00	15.0	20.0
		24	0.41	1.23	1.64	12.3	16.4
20	6.0	12	1.04	3.12	4.16	15.6	20.8
		15	0.89	2.67	3.56	13.4	17.8
		18	0.79	2.37	3.16	11.9	15.8
		24	0.65	1.95	2.60	9.7	13.0
40	12.0	12	1.65	4.95	6.60	12.4	16.5
		15	1.42	4.26	5.68	10.7	14.2
		18	1.26	3.78	5.04	9.5	12.6
		24	1.04	3.12	4.16	7.8	10.4

gutter at the control. The two plants illustrated are among the larger plants in the United States. Many plants have lateral wash-water gutters 10 ft or less in length. If $l = 10$ ft and $b = 12$ in., with $q = 0.3$ cu ft per sec per foot of gutter, the quantity of water entering after the control is between 19.5 and 26.0 per cent. The greater the length for a given increment of flow, and the larger the value of b, the more nearly correct is the author's assumption. Naturally, the greater the rate of washing, the greater will be the value of d_c. Although the author's assumption is fundamentally unsound, it provides a means for calculating an artificial but apparently reliable starting point for the water-surface curve.

With the foregoing facts and a simplification of Equation (29), design computations may be simplified to a less general, but easier, method than that proposed by the author.

If

$$2 S x_c \bar{d} = \frac{f x_c d_c^3}{12 \bar{R} \bar{d}} \quad \ldots\ldots (44)$$

Equation (29) becomes

$$H_0 = \sqrt{3}\, d_c \quad \ldots\ldots (45)$$

and there appears to be no reason why this condition cannot be obtained for the usual channel, since physically it implies giving the channel slope enough to compensate for the assumed friction. From Equation (44) there results a unique and simple relation between S and f which permits the designer to observe the effect of the assumed f upon the slope of the channel. This relation is

$$\frac{S}{f} = \frac{d_c^3}{24 \bar{R} \bar{d}^2} \quad \ldots\ldots (46)$$

and may be placed in more practical form by the following relations

$$\bar{d} = \frac{H_0 + d_c}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47a)$$

and

$$\bar{R} = \frac{\left(\frac{H_0 + d_c}{2}\right) b}{H_0 + d_c + b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47b)$$

which are approximate, but permissible when the uncertainty in the selection of f is considered. Insertion of these values in Equation (46) results in

$$\frac{S}{f} = \frac{d_c^3}{3}\left[\frac{H_0 + d_c + b}{(H_0 + d_c)^3\, b}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

which, with Equation (45), and the critical-depth formula, Equation (28), allows quick calculation of the principal dimensions of the spillway channel.

Equations (45), (48), and (28) may be used by the designer to give values for the size of lateral wash-water troughs which, while approximate, are as accurate as the value of "f" which must be assumed.

In the design of a system of wash-water troughs it is first necessary to assume a layout (as dictated by practice) and then investigate that particular layout. For any spacing of gutters there are an infinite number of breadths of gutter which can be used, but experience has shown that the wash-water gutters should not occupy more than one-third of the surface area. The applicability of Equations (45), (48), and (28) to design is given in the following illustrative problem.

The wash-water troughs are 20 ft long, spaced 6 ft on centers, and designed to carry a wash-water rate of 36 in. vertical rise per minute. In the illustrative

TABLE 4.—COMPARISON OF APPROXIMATE DESIGN DATA FOR TWO, TWENTY-FOOT, WASH-WATER TROUGHS

Width of trough, b, in feet	Flow		Depth, d_c, in feet	Head, H_0, in feet	Ratio, $\frac{S}{f}$	Slope, S	
	q, in cubic feet per second per foot	Q, in cubic feet per second				For a friction factor, $f = 0.030$	For a friction factor, $f = 0.10$
1.0	0.30	6.0	1.04	1.80	0.063	0.0019	0.0063
2.0	0.30	6.0	0.653	1.13	0.0312	0.00094	0.00312

computations a breadth of 12 in. was used, but Table 4 gives results for a breadth of 24 in. also. The inflow, q, per lineal foot of trough is given by $q = \frac{36}{60 \times 12} \times 6 = 0.3$ cu ft per sec per ft; the total flow Q is equal to $0.3 \times 20 = 6.0$ cu ft per sec; from Equation (28) $d_c = \sqrt[3]{\frac{(6.0)^2}{32.2 \times 1^2}} = 1.04$ ft; and, from Equation (45) $H_0 = \sqrt{3} \times 1.04 = 1.80$ ft. Substituting in Equation

(48), $\frac{S}{f} = 0.063$. It is now necessary to assume a value of f. Many more experimental data are necessary before this value can be predicted accurately; but values of f of 0.03 and 0.10, from the author's data, will require respective slopes of 0.0019 and 0.0063.

The foregoing computations are summarized in Table 4, which includes for comparison the results obtained from the same filter when a value of $b = 24$ in. is assumed. It is interesting to note that the value of S for a given value of b and f varies quite widely. A variation of f from 0.030 to 0.10 requires an increase of slope from 0.0019 to 0.0063 for a 12-in. width and from 0.00094 to 0.00312 for a 24-in. width. As would be expected from Equation (48), for all other items constant S varies directly with f; that is, the slope required for $f = 0.10$ for either a 12-in. or a 24-in. width is 3.33 times the slope required for $f = 0.03$. It is granted that these simplifications of the author's formula give only approximate results; but they are as accurate as the values of f. There is need for further research to determine the value of f for short channels, particularly on short wash-water lateral gutters and trickling filter collection troughs.

Presumably the experimental channel at Massachusetts Institute of Technology did not have free outfall according to the surface curve of Fig. 6.

F. KNAPP,[10] ESQ. (by letter).[10a]—This paper, treating a special chapter of applied hydraulics, should command the interest of all engineers interested in problems involving the flow of water in open channels. The author's treatment of the question is so complete that scarcely any improvement is possible. The same problem is being encountered in the design of tail-races for hydro-electric plants with laterals connecting the main channel with the turbine pits.

The author mentions a formula based upon the assumption that the draw-down of the water surface is equal to the velocity head. Rearranging Equation (17) and using the Manning flow formula instead of the Weisbach equation, the following is obtained:

$$dy = \frac{V^2}{g}\left(\frac{g}{K^2 R^{\frac{4}{3}}}\,dx + \frac{dQ}{Q} + \frac{dV}{V}\right) \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

in which $dQ = q\,dx$. The roughness factor K of Manning's formula is approximately equal to the reciprocal of Kutter's "n"; and R is the hydraulic radius of the spillway channel.

Equation (49) indicates the causes for the draw-down of the water level—namely, friction, increase of discharge and change of velocity. A comparison of this formula with the more general expression derived by H. Favre [11] shows that the derivation neglects the component of the velocity of the lateral discharge parallel to the axis of the flume, as well as the non-uniform velocity distribution of the flow in the channel. However, only in exceptional cases

[10] São Paulo, Brazil.

[10a] Received by the Secretary April 12, 1939.

[11] "Contribution a l'Étude des Eaux Courants," by H. Favre, Editor Rascher & Cie., Zurich, Leipzig and Stuttgart, 1933, Equation (14), p. 17.

need these factors be taken into account and for this reason Equation (17) is perfectly general.

Equation (29) is only valid for the case with critical depth at the downstream end of the spillway channel. Neglecting, for the moment, the slope S of the channel and the friction, Equation (29) may be written:

$$H_0 = \sqrt{3\, d_c^2} = d_c \sqrt{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (50a)$$

Note the comparison with the rearranged Equation (23):

$$H_0 = d \sqrt{2 \left(\frac{d_c}{d}\right)^3 + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (50b)$$

Equation (50*b*) is valid in all cases, including the freely discharging flume.

The author calls attention to the difference between lateral spillway channels and side weirs. As shown by Mr. Favre,[12] Equation (17) is also valid for the case of a side weir provided the term $\frac{dQ}{Q}$ is disregarded. Therefore:

$$dy = \frac{V^2}{g} \left(\frac{g}{K^2 R^{\frac{4}{3}}} dx + \frac{dV}{V} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

This equation is exactly equal to the well-known expression for the computation of back-water curves. However, there is a fundamental difference between the two formulas. The condition of continuity in the case of the back-water curve is:

$$V_2 A_2 - V_1 A_1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (52a)$$

and in the case of the side weir:

$$V_2 A_2 - V_1 A_1 = \Delta Q \quad \ldots\ldots\ldots\ldots\ldots\ldots (52b)$$

in which A is the cross-sectional area of the canal and ΔQ is the discharge over the side weir.

The model tests conducted by Mr. Favre [13] at the "Laboratoire des Recherches Hydrauliques Annexé à l'École Polytechnique Fédéral de Zurich" have shown that in the case of tail-race channels for hydro-electric power stations, it becomes necessary to apply Equation (17) neglecting the friction term. This procedure counterbalances, partly, the error due to the velocity component of the lateral flow, which is always present.

The smooth surfaces of the experimental profiles, given by the author, represent a most surprising fact. Observations made on lateral spillway channels for dams always indicate a very wavy profile, the theoretical surface of the water passing through the centers of these waves.

The author is to be commended for his most interesting, useful, and timely paper.

CARL ROHWER,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—The applicability of the momentum principle to the general solution of the problems arising in

[12] "Contribution a l'Étude des Eaux Courants," by H. Favre, Editor Rascher & Cie., Zurich, Leipzig and Stuttgart, 1933, p. 60.

[13] *Loc. cit.*, p. 51.

[14] Irrig. Engr., Bureau of Agri. Eng., U. S. Dept. of Agriculture, Colorado Experiment Station, Fort Collins, Colo.

[14a] Received by the Secretary April 14, 1939.

the design of side channel spillways has been demonstrated by Mr. Hinds[2] and Mr. Favre.[3] The analysis presented in this paper shows the application of the principle to the special case of the design of wash-water gutters, with a consequent simplification in the procedure.

The experimental data presented by these investigators show that the calculated water-surface profiles are in reasonably close agreement with the observed profiles. This fact seems remarkable in view of the assumptions that must be made in order to simplify the theoretical analysis. For example, to compute the momentum at any section in the channel, it is necessary to know the average velocity of the water at the section. It is assumed to be equal to the discharge at the section divided by the area. Actually, the velocity in any section may vary through wide limits; some of the velocities may even be negative. It is evident, therefore, that the average velocity is greater than the mean computed in this manner. Since the average depth of the water in the channel is used in computing the velocity, entrained air which increases the indicated depth will tend to increase the difference between the average and the mean velocity still further. The total difference will depend on the depth of water flowing into the channel and on the turbulence produced.

The photographs presented by Messrs. Hinds and Favre show that the water flowing over the spillway crest creates a violent disturbance in the water, and may produce a definite transverse roll. The writer has observed spillway models in operation, and although the depth of water flowing over the weir into the channel did not exceed 6 in. in these model studies, the agitation of the water was pronounced. From the foregoing discussion, it is apparent that there are important differences between the assumed and observed conditions. There may be compensating factors, however, which tend to minimize the effect of the errors in these assumptions.

In evaluating the friction factor "*f*" (see heading "Experimental Evaluation of Friction Factor") by substituting in Equation (23) the channel depths observed during tests made on several spillways, the author found values ranging from 0.033 to 0.11; and, in one case the value of "*f*" was negative. The wide range in the friction factors leads the writer to believe that part of the variation is due to the fact that the formula does not agree precisely with the experimental data and that, in effect, the friction factor becomes a coefficient that takes care of the difference between the observed and computed data as well as the friction in the channel.

The use of a friction factor derived in this manner in computing the depths for the profiles (Figs. 4, 5, and 6) by substituting in the formulas would tend to make the computed results agree more closely with the observed results than the accuracy of the formulas warrants. In designing a channel for a wash-water gutter, it would be necessary to assume a value of "*f*"; and consequently the computed profiles might not agree so closely with those observed in the actual structure. It would have been helpful if the author had included in this paper a table of values of "*f*" for use by any one having to design a spillway of this kind.

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[2] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 885.
[3] *Engineering News-Record*, October 25, 1934, p. 520.

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DISCUSSIONS

THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS

Discussion

BY MESSRS. C. MAXWELL STANLEY, AND FRANCIS J. MORGAN AND FREDERICK C. ZEIGLER

C. MAXWELL STANLEY,[10] ASSOC. M. AM. SOC. C. E. (by letter).[10a]—The outline of the legal aspects of the risks involved in sub-surface construction, as presented by Mr. Herwitz, should be of help to every engineer who prepares specifications for such work. Although most engineers may have avoided litigation arising from unexpected conditions under the surface of the ground, few have avoided controversy with contractors, which has stopped short of litigation only because of compromise, extra compensation, or because the sum involved was too small to justify legal action.

As emphasized by Mr. Herwitz, there must be a complete "meeting of minds" between the parties to the contract if confusion and misunderstanding are to be avoided. Such a "meeting of minds" requires a full understanding, on the part of both parties to the contract, of the risks and the responsibilities to be assumed by each. Such an understanding can be obtained only through the contract documents, and, more particularly, through the specifications and the plans prepared by the engineer. Many of the cases cited by Mr. Herwitz obviously involve specifications in which the provisions regarding the risks and responsibilities have been vague or indefinite, thus preventing a "meeting of minds." In such cases, it is natural that disputes will arise which create controversy, and which may lead to litigation.

Every engineer should recognize that there are hazards involved in sub-surface construction because experience has shown that no matter how carefully subsoil investigations are made, conditions may be encountered that are different from those anticipated. Likewise, every experienced engineer should realize that various types of subsoil require different methods of construction and

NOTE.—This paper by Oren Clive Herwitz, Esq., was published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939 by Messrs. Alonzo J. Hammond, Frederick W. Newton, David A. Molitor, F. B. Marsh, and Evan S. Martin.

[10] Cons. Engr., Young & Stanley, Inc., Muscatine, Iowa.

[10a] Received by the Secretary March 20, 1939.

types of equipment and, hence, result in different costs of construction. Furthermore, he should realize that if a contractor is required to assume the risk of unexpected subsoil conditions, that contractor is entitled to compensation for this risk and must include in his bid a sum sufficient to cover the item.

In preparing his specifications, the engineer has the three following choices of placing the responsibility for unexpected subsoil conditions:

(1) The entire responsibility may be placed on the contractor who must thus assume all risks and who will, of necessity, protect himself by including in his bid a sum which he estimates is equal to the risk that he must carry, plus the cost of any borings or investigations which he may make.

(2) The responsibility may be assumed by the owner by giving information on subsoil conditions and by guaranteeing this information. In this case, the risk is entirely upon the owner and the contractor need not protect himself by covering these items in his bid and, hence, may bid lower.

(3) The responsibility may be divided between the contractor and the owner. In such cases, the owner may provide available data on subsoil conditions but require the contractor to interpret them and be responsible for any misinterpretations or for any conditions which prove to be different from those indicated. Such a case saves the contractor the expense of borings but still makes him assume a risk for which he is entitled to proper compensation.

There are numerous circumstances which fully justify either of the first two choices, and there are undoubtedly some cases where the third choice is proper. It would seem, however, that the greatest difficulty has arisen in those cases in which the engineer has endeavored to "straddle the fence" and divide the responsibility between owner and contractor and has failed to state in the specifications, clearly and definitely, just what is intended. Sometimes the engineer has realized the shortcomings of the data he has available on subsoil conditions and, although he has used these to outline the general methods of construction and type of work on the project, he has endeavored to avoid the assumption of responsibility on the part of the owner by placing it upon the contractor through means of general or "catch-all" phrases in the specifications.

After the decision has been made as to who shall bear the responsibility for any unforeseen conditions, the specifications must then be prepared in such a manner that there will be a complete "meeting of minds." Vague and indefinite provisions and so-called "catch-all" phrases which are inserted in an effort to throw responsibility on the contractor while, at the same time, the engineer is endeavoring to specify methods and equipment, have no place in a specification. The engineer should state, fully, clearly, and fearlessly, who is to be responsible for unforeseen ground or subsoil conditions. If the owner is bearing the risk, the contractor should be instructed to bid upon the subsoil conditions as outlined in the data included in the specifications, and he should be compensated for unforeseen conditions which may change the cost of construction. Often, this can be taken care of by the use of unit prices on different types of material that may be encountered. On the other hand, if the contractor is to bear the responsibility, he should be made aware of it so that he may protect himself to whatever extent he deems necessary when he bids

upon the work; and then, if unforeseen conditions are encountered, the specifications will be sufficiently clear and precise so that he has no valid claim for any additional compensation.

In all events, however, the specifications must be clear and precise and must deal fairly with both parties; and, they must bring about a complete "meeting of minds."

FRANCIS J. MORGAN,[11] ESQ., AND FREDERICK C. ZEIGLER,[12] M. AM. SOC. C. E. (by letter).[12a]—In the presentation of his paper, Mr. Herwitz has made a valuable contribution to engineers and others engaged in the preparation of construction contracts and their appurtenant specifications, plans, and documents where underground work is involved.

Sub-surface exploration work, in some cases (such as for tunnels and pipe lines) may be done primarily for the information of the engineer to guide him in establishing the location, grade, type, and shape of his structure. The character of materials and conditions encountered in borings may require radical changes in the engineer's preliminary designs so as to make these borings quite incomplete, if not unreliable, when used by contractors for bidding purposes. Sometimes complete deep borings are difficult to make especially in glacial drift where boulders abound and blasting is restricted in public thoroughfares. Drilling progress under such a condition is likely to be very slow with consequent long delay in letting the construction contract.

Generally, the engineer on public work tries to obtain the best possible sub-surface data. If he employs a reliable and experienced boring contractor, it is obvious that the bidding contractors on the subsequent construction work have available as much information as to the nature of the sub-surface conditions as he has. Of course, the engineer is on "dangerous ground" when he attempts to interpret sub-surface conditions and draws on his own imagination as to the kind, form, and types of constructions or configurations of strata which the eye cannot see.

It should be the function of the engineer to protect, with great care, the interests of the owner or municipality for which construction work is to be done so that damage claims will be avoided or, if taken to Court, will not result in costly judgments. Waivers alone appear in certain cases to carry little weight in Court.

Since much of the public bidding on construction jobs is clearly a gamble on the part of many contractors, it behooves the engineer to use extreme care to make the provisions of his contract documents as tight as possible to protect the owner, especially if borings and other records are made available to bidders. It is unfortunate, but it must be admitted, that there are a few contractors who are quite ready to use, against the engineer, any honestly prepared informatory data that have been supplied to him in advance of bidding, especially where the contractor has been the loser financially through his own mis-

[11] Deputy, County Attorney, Westchester County, Yonkers, N. Y.

[12] Formerly Project Engr., Westchester County Sanitary Sewer Commission; and Project Engr., Bureau of Sewage Disposal, Dept. of Public Works, New York, N. Y.

[12a] Received by the Secretary March 15, 1939.

management or otherwise. The engineer must be prepared for that kind of a contractor.

It should be realized that on public work, the contractor usually can secure a Court order to make a "fishing expedition" to examine all the engineer's records for evidence to use against him in the Trial Court.

It is realized that it would have taken much space if the author of the paper had covered, in reasonable detail, the various Court Citations mentioned in the Appendix. Some of them, however, would probably be of considerable value to the engineer who is charged with the preparation of important contracts because the average engineer has little or no true conception of the judicial viewpoint.

In a 10-yr construction program in Westchester County, New York, the case of Montrose Contracting Company, Incorporated *vs.* County of Westchester (66)[12b] was the only one of several claims taken to Court by contractors, which finally resulted in a judgment in the contractor's favor. Bidders generally recognized that the work, a tunnel, was "a wet job," in a valley at considerable depth below the bed of a near-by stream. Sufficient information was available in the contract documents and borings to put the bidders on notice that the work was without doubt very wet. It is difficult, at least for a layman, to understand how a Court could be convinced that the contract documents, which clearly delineated wet conditions and required an examination of the site by bidders, meant anything else to experienced contractors than the necessity of using accepted methods under such circumstances to keep the work dry during construction.

In the first trial, the contractor contended that the contract provisions for under-drains, alignment holes, ventilation, care of water and placing of concrete tunnel lining in not less than 12-ft sections all indicated a free air job with the exception of about 600 ft of tunnel and that they could not be performed where compressed air was used. No defense was offered and the judge granted the County's motion for a directed verdict in its favor (80). The Circuit Court of Appeals on an appeal by the contractor reversed the judgment entered in favor of the County and ordered the second trial holding that as a matter of law the under-drains could not be built in compressed air; that the alignment holes required were used in free air but never in compressed air; that the specifications relating to ventilation, concreting and care of water were suitable for a "free-air" job and impossible of use in a compressed-air job.

Regardless of the fact that the County in the second trial offered expert evidence to show that the aforementioned provisions could be used in compressed air as well as in free-air tunneling the opinion of the Circuit Court prevailed and the contractor was successful in recovering a substantial judgment. The County then took an appeal to the Circuit Court of Appeals but was unsuccessful (81). The Supreme Court of the United States refused to grant Writs of Certiorari on both applications therefor made by the County, after each appeal had been decided by the Circuit Court of Appeals.

[12b] For reference to numerals in parenthesis, see "Court Citations and References," in the Appendix of the paper and at the end of this discussion.

The lawsuit referred to herein presents a very interesting problem both for lawyers and engineers in that it relates to the interpretation of a contract, where mixed questions of law and fact are involved.

It is the duty of the Courts to interpret contracts and explain their meaning. The Trial Court on the first trial held in substance that the contract did not represent and impliedly warrant the tunnel job to be substantially a "free-air" job. That judge apparently interpreted the contract as it was written and especially those parts that referred to the use of compressed air. The words which related to compressed air, and the partial payment for 600 ft, specifically said "regardless of whether more be used or not" (82).

In its opinion on the first appeal the Circuit Court concluded that the compressed air provisions of the contract could not negate the representations in the plans that the tunnel was primarily one to be constructed in free air. It held further that the compressed air provisions covered a variation, but not a transformation, and that they would be effective if somewhat more than 600 ft of work were required in compressed air. It did not say how much more.

Furthermore, the Circuit Court took the evidence offered by the contractor and, with no evidence offered by the County, interpreted the contract to mean that the provisions of the contract relating to under-drains, alignment holes, ventilation, pouring of concrete in not less than 12-ft sections and the care of water applied to free air, and were impossible of use in compressed air.

If it is assumed that the trial judge on the first trial erred on the law in interpreting the contract, then questions of fact arose as to whether or not the aforementioned provisions of the contract applied both to free air and compressed air, and whether or not these things could be done only in free air and could not be done in compressed air.

As the case was tried before a jury, it seems that these questions, after the evidence thereon had been offered by both sides in the second trial, should have been given to the jury to determine as questions of fact. If this had been done and the jury had determined that these provisions of the contract could be performed only in free air, then the Court would have been in the position to apply these facts to the contract and hold that the contract represented, and impliedly warranted, the tunnel to have been substantially a "free-air" job.

Viewing it from the other side and assuming that the jury found these provisions of the contract could be performed in compressed air as well as in free air, the Court would have to say that there was no representation and implied warranty that the work was substantially a "free-air" job.

In his first opinion, did the Circuit Court judge stray from the beaten paths of the law and wander into the realms and fields of engineering and thereby become an engineer as well as a judge? Who determined the fact that the under-drains or the alignment holes could not be built in compressed air, or the ventilation provisions could not be used in compressed air, or that the care of water provisions was impossible of use in compressed air, or that concrete in at least 12-ft sections could not be poured in compressed air? It is a fundamental principle of law that the duty of a common-law jury is to

determine and decide questions of fact. Furthermore, another question suggests itself and that is: Did the contract contain an "express warranty" that fixed the sum of money the County would pay specifically and directly for compressed-air work, and thereby require the contractor to include in Item 1 of the bid (this was a lump sum contract) any additional sum it needed to compensate itself for all compressed-air work costing more than $30 a foot, no matter how much was done under compressed air?

Different technicalities seem to arise in every Court case, which affects its outcome. Even the viewpoints of the Courts appear to differ; the Westchester case in point has no parallel. Care was taken to avoid specifying both the methods of construction and the results to be attained but the Court may have thought otherwise. A distinction has been drawn between constructional features which serve a useful purpose in the completed structure and those useful only during construction whether the engineer requires them for his use or not; such as, for instance, the alignment holes for checking tunnel lines from the ground surface.

Because of sub-surface conditions which a bidder seems to have disregarded this discussion shows what can happen in litigation regardless of the care in the preparation of borings and a seemingly fair and reasonable contract. It supports the author's conclusion that each case must rest upon its own distinctive facts. It appears, however, that the safest contract documents are those whose provisions are very general so far as possible; otherwise every conceivable contingency must be covered.

Court Citations and References.—

(80) 80 Fed. (2d) 841.

(81) 94 Fed. (2d) 580.

(82) The contract contained the following provisions concerning the use of compressed air:

> "The Contractor hereby agrees to receive and accept as and for his full compensation for furnishing all the labor, plant and materials for, and building, constructing and fully completing and maintaining for 12 months all the work called for under this contract and for all the labor, plant and materials incidental thereto as required by the specifications and as shown on the plans of the work referred to therein, the following prices:
>
> 78-inch sewer in tunnel complete as indicated on the plans and specifications of this Contract, including shafts, portals, boat hole, manholes, alinement holes, sewer connections and all their appurtenances; except only any partial compensation for compressed air if used as listed in item 2 below; @ Nine hundred sixty-seven thousand and 00/100 dollars.
>
> ($967,000.00) lump sum Amount $967,000.00

Maximum partial compensation for labor, plant and materials under compressed air when and if actually used as specified but not in excess of 600 feet in length of tunnel @ Thirty and no/100 dollars. ($30.00) per foot Amount $18,000.00

Total sum bid in words Nine hundred Eighty-five thousand and 00/100 dollars.

Total Amount $985,000.00

"The above total includes all excavation, backfilling, waterproofing, masonry, concrete, reinforcement, forms, pipe, underdrain, fittings, sheeting, shoring, forepoling, timbering, piling, foundations, testing, maintenance, labor, plant, materials, insurance and all other things shown, specified or necessary for the full, watertight and enduring completion of the entire work.

* * * *

"That where material other than solid rock is encountered, containing loose sand or mud, which shifts with water contained in it, and the Contractor chooses to use compressed air to pass through it; in order to partly relieve him of the additional cost of preventing and caring for the incoming water and material or restraining it, an allowance will be made throughout the stretch where compressed air is actually used under the above conditions of $30.00 per foot and for not more than 600 feet of length, where actually used, regardless of whether more be used or not."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THEORY OF LIMIT DESIGN

Discussion

BY MESSRS. JOHN H. MEURSINGE, I. K. SILVERMAN, EDWARD GODFREY, BASIL SOUROCHNIKOFF, E. MIRABELLI, C. M. GOODRICH, GEORGE WINTER, AND FRANCIS E. SIMPSON

JOHN H. MEURSINGE,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—Roughly, this paper can be divided into three parts: (1) A criticism of the structural engineers' conventional design methods; (2) a treatise on the theory of limit design, with examples; and (3) a review of the applications of the theory, with references to the proper authorities. All three parts will be discussed in the order mentioned.

(*1*).—In the light of present-day conditions, the harsh criticism of the structural engineers' "love" for stress analyses seems unwarranted because, since the discoveries of Galilei, "all" professions of the civilized world have adopted a number of more or less mathematical rules by means of which the object of their observation is tested. The structural engineer is no exception. Physicians satisfy their urge for numerical terminology by reading thermometers and blood pressure apparatus, and by counting heart beats and white and red blood corpuscles; and economists measure the economic health of the nation by the analysis of the balance sheet. Civilized Man has forgotten entirely that all other living creatures on earth live, multiply, do their engineering work, and die, without worrying about numbers. Their entire existence is based on the application of the laws of equilibrium, efficiency, and usefulness. Nevertheless, the status of their health, economics, and the strength of their structures (as, for instance, the bees) can be perfectly well expressed in terms other than mathematics. If their health is vigorous they carry capacity loads; if their economic system is functioning properly, each member of their society will receive capacity supplies; and, when their engineering structures are in good working order, they will carry capacity loads.

NOTE.—This paper by J. A. Van den Broek, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[15] Huntington Park, Calif.

[15a] Received by the Secretary March 8, 1939.

As soon as Mankind adopts such a system, in which his accomplishments are judged by tangible results, the engineer can begin to ignore mathematics. In the meantime, he had better follow the methods and conventions of the society in which he happens to live. Any other course would impair his usefulness seriously.

(*2*).—In the theory of limit design the investigator takes advantage of the ultimate strength of the full cross-section, even when a structure is subjected to bending stresses. There is no more efficient application of the material; and, as a result, structures are obtained, the strength of which can be expressed in capacity loads—that is, in terms of usefulness. Thus far structural designs have merely satisfied a number of mathematical requirements; hence, the theory advanced in this paper is a step in the right direction. The engineer should keep in mind, however, the fact that Hooke's law and the theory of limit design are both "simplifications of a very intricate reality—nothing more nor less. Both may be compared to cartoons, because they both 'express reality in a very primitive matter.'"[16]

Professor Kist taught the writer how to apply the theory of limit design; and, although the examples in the paper had to be quite elaborate in order to clarify the principles for the benefit of new readers, the application can be simplified in practice for the greater part of the problems which the average designer has to solve. For instance, in Case 1 one needs simply to compute the maximum moment of resistance; then immediately draw the moment diagram of Fig. 4; and, apply the static equilibrium formula:

$$w' \frac{L}{2} \times \frac{L}{4} = 2\,M_1 = 60\,000$$

and solve: $w' = 13\,333$ lb.

For concentrated loads: Draw the moment line for capacity loads shown in Fig. 17. Considering the structure as a simple beam with maximum moments of $2\,M$, and calling the end reactions R_1 and R_2: $R_1 = \frac{2\,M_1'}{a}$; and, $R_2 = \frac{2\,M_1}{c}$. By assuming different values for P_1, one can determine the complementary capacity loads P_2.

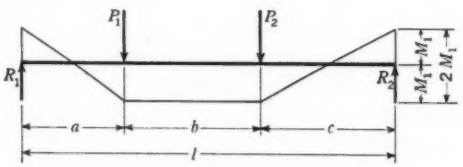

FIG. 17

Professor Van den Broek states that he acquires a factor of safety of 2; but in reality it can be called 3 because Fig. 4 represents a condition in which the cross-section at the middle of the beam still has a triangular stress-distribution diagram. Now, according to the ductility philosophy the beam cannot collapse until two sections have rectangular diagrams; and, as Professor Van

[16] "Ductility as a Base for Design Computations of Steel Bridges and Structures Instead of Proportionality of Stress and Strain," by N. C. Kist, International Congress for Metallic Structures, Liége, Belgium, 1930.

den Broek demonstrates in Equation (2), this condition means a 50% increase in strength for rectangular beams.

This simplification of Case 1 is also helpful in solving Case 4, the continuous beams shown in Figs. 9 and 10. First, find the moment of resistance: $\frac{I}{c} = 30$; $s = 30\,000$; and, $M_1 = 30 \times 30\,000 = 900\,000$ in-lb = 75 000 ft-lb. Draw the moment line of Fig. 9(b). Knowing the location of P_1, the computer finds for the left beam reaction: $\frac{75\,000}{5} = 15\,000$ lb. Taking moments about the center support: $15\,000 \times 10 - P_1 \times 5 = -75\,000$; or, $P_1 = 45\,000$ lb.

At first the application of the theory of limit design seems most simple as applied to structures that have no bending stresses, according to the elastic theory method (as, for instance, the tower in Fig. 15); but, if Professor Van den Broek had demonstrated the new method of detailing gusset plates, he would have revealed another radical departure from conventional methods.

(*3*).—The author considers that the problem of riveted connections is a poor illustration of the application of the theory of limit design (see "Design of Riveted Connections"). For engineers who have had little experience in modern design methods this section happens to be quite convincing. It reveals, plainly, the important fact that designers have to take advantage of ductility (but have not spoken or written about it) in many details, at the same time ignoring it in the design of the main members.

This same chapter, unfortunately, contains a statement from which the reader may infer that the theory of elasticity does not fail as readily in welded connections as in riveted connections. Professor Van den Broek states:

> "Nevertheless, the theory of elasticity fails as completely in the design of rivet connections as it does in the analysis of many other redundant structures.
>
> "Rivet connections appear to be 'giving ground' to welded joints. It may be hoped that they soon will disappear entirely."

H. M. Priest, M. Am. Soc. C. E., has presented[17] stress curves for both riveted and welded connections. In the specimens tested the ends of both connections were over-stressed, and the welded connection would not have held better than the riveted connection except for the ductility of the material.

Connections play an important rôle in structures designed in accordance with the author's theory. Whereas, in structures designed to conform with Hooke's law, the connections are often made lighter than the connected member, the theory of limit design does not tolerate such a condition. Consequently, the writer would add to the last paragraph under the heading "Practical Application": "* * * are guarded against, and as long as all connections are capable of transferring the elastic-limit stresses that may appear in the heaviest connected member."

What about the future of the theory of limit design? An interesting experience of the writer has a bearing on this question. In 1922, while he was a student at the Technical University of Delft, Netherlands, the writer was admonished by Professor Kist for using the elasticity theory of a continuous beam

[17] "The Practical Design of Welded Steel Structures," by H. M. Priest, *The Welding Journal*, August, 1933, Fig. 8, p. 9.

on three supports in order to determine the size of three rollers beneath a shallow roller bearing. Professor Kist demonstrated that the short distance between the actual bearing and the rollers did not give the bearing force in that type of support an opportunity to spread to the two outside rollers until the compression stresses in the middle roller had reached the elastic limit. In this case the theory of limit design offered a more logical solution.

In 1937, the writer, then in the employ of an oil company, was called upon to design a shallow roller bearing beneath a horizontal pressure tank. Remembering his college days, he began with the theory of limit design; but this computation was rejected by the engineer in charge of the design as being contrary to "standard practice." No other action was to have been expected. Hooke's law has been the conventional standard for 250 yr; the new theory is a radical departure from that method. Neither has the new philosophy either established a convenient inter-relationship with modern designers' general process of thinking. Furthermore, the structural engineer's brains resemble a carbureter. After it has been set for a certain mixture of assumptions, it is difficult to adjust it to a new recipe, even if the latter may prove to be more economical.

Hence the writer has concluded that the theory of limit design will become accepted only gradually. Its extensive application will be postponed until the civilized world has learned that the health and happiness of mankind, as well as the strength of structures, can be expressed in terms of usefulness instead of in mathematical symbols.

I. K. SILVERMAN,[18] Assoc. M. Am. Soc. C. E. (by letter).[18a]—The simplest structural form is a straight bar loaded axially. An example is the standard test piece used in determining such quantities as proportional limit, yield point, and ultimate stress. Numerical values of these quantities, obtained from tests on this type of structure, are the bases upon which the proportions of structural members, however they may occur, are estimated.

The proportional limit is defined as that stress at which stress and deformation are no longer connected linearly. The yield point has been defined as that stress at which there is a marked extension with very little increase in load. Sometimes this value is defined as the stress at which a certain permanent strain is obtained. The ultimate stress is determined by dividing the maximum loading recorded in a test by the cross-sectional area of the test piece.

When inserted in codes, these values (which, fundamentally, hold only for the simple case of an axially loaded bar) control the design of as simple a structure as a bar in tension or the relatively complex structure exemplified by a single-span beam.

Assume that instead of the standard test piece a single-span beam of given material, under a given type of loading, is taken as a standard. Then it is possible, in the light of the foregoing definitions, to obtain definite quantities corresponding to proportional limit, yield point, and ultimate strength. It will be found that these values are influenced slightly by the type of loading initially chosen (uniform or distributed) but are greatly influenced by the form

[18] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[18a] Received by the Secretary March 10, 1939.

of cross-section.[19] E. Mirabelli,[20] M. Am. Soc. C. E., has shown analytically, and has supported his results by citation of tests, that the factor of safety of a beam, based on its own yield point, may be as much as four, compared to a factor of safety of two, based upon the yield point of a simple tensile-stress test.

Although it seems impracticable to test every type of structure to obtain its proportional limit, yield point, etc., this point of view should lead to a new philosophy of design, as the author states.

The results of various tests on standard tensile specimens have led to far-reaching results in mechanical, structural design. It is well known that if a mild steel is loaded in tension above its yield point, unloaded, and then subsequently reloaded, a raising of the yield point is obtained. Along with this raising of the yield point in tension, there is a subsequent lowering of the yield point in compression. This result is known as the Bauschinger effect. In explaining these results, H. Hencky,[21] strangely enough, called upon the phenomena which the author has described. Mr. Hencky's analogy consisted of a three-membered symmetrical truss (Fig. 18(a)), statically indeterminate in the first degree. While the load is $O \leqq P \leqq 2\, s_1 A$ (s_1 is the yield point stress in simple tension) an elastic state of stress exists. The load-deformation curve is Line DF (Fig. 18(b)). At the load $P = 2\, s_1 A$ the vertical bar DC (Fig. 18 (a)) becomes plastic and, assuming that the material has a marked yield point, this vertical bar will not increase in stress as the load is increased to, say, $3\, s_1 A$. The rate of deflection will be greater, as shown by the slope of $F\,F'$ (Fig. 18(b)). If the structure is unloaded, a permanent deformation corresponding to $D\,D'$ will occur, with residual stresses of tension in Bars $A\,D$, $B\,D$ and compression in Bar $D\,C$ (see Fig. 18 (a)). If the load is reversed the yield point will be reached at a lower load as shown by $F'F''$ because of the residual compressive stress in Member $D\,C$. However, if the structure is reloaded in the same direction as the original force, the load will increase along $D'F'$ linearly and will follow along $F'F''$ until all the bars have reached the yield stress s_1. This occurs at a value of the load $P = 5\, s_1 A$. Beyond this load, deflections outside the range of magnitude of elastic deformations are to be expected. From the point of view of the theory of elasticity the load capacity of the structure would be taken as $2\, s_1 A$. Using the plastic properties of the material the load capacity could be taken as $5\, s_1 A$.

The properties given by the simple tensile test, and which have their counterpart in the behavior of the simple structure in Fig. 18(a), have been used advantageously by mechanical engineers. Advantage has been taken of the yielding property of the material and its consequent higher yield point in the design of chains; the favorable residual stresses resulting from permanent deformation have been used in the design of rotors;[22] over-strain of metals has

[19] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1397.

[20] *Loc. cit.*, p. 1400.

[21] "Zum Theorie Plastischer Deformationen und der hierdurch im Material Hervorgerufenen Nachspannungen," by H. Hencky, *Zeitschrift für angewandte Mathematik und Mechanik*, 1924, p. 323; also, "Strength of Materials," by S. Timoshenko, Part II, 1930, p. 668.

[22] "Zum Theorie Plastischer Deformationen und der hierdurch im Material Hervorgerufenen Nachspannungen," by H. Hencky, *Zeitschrift für angewandte Mathematik und Mechanik*, 1924, p. 331; also "Stress Distribution in Rotating Disks of Ductile Material After the Yield Point Has Been Reached," by A. Nádai and L. H. Donnell, *Transactions*, A. S. M. E., Vol. 51, 1929, APM–51–16, p. 173.

created residual stresses that are favorable during the firing of guns. It is to be noted that in all these cases the direction of the loads is fixed so that the question of reversal does not enter.

The preoccupation of engineers with the concept of stress, as the author states, can be traced to the definition of "factor of safety" as given in textbooks on strength of materials. There, the "factor of safety" is defined as the ratio

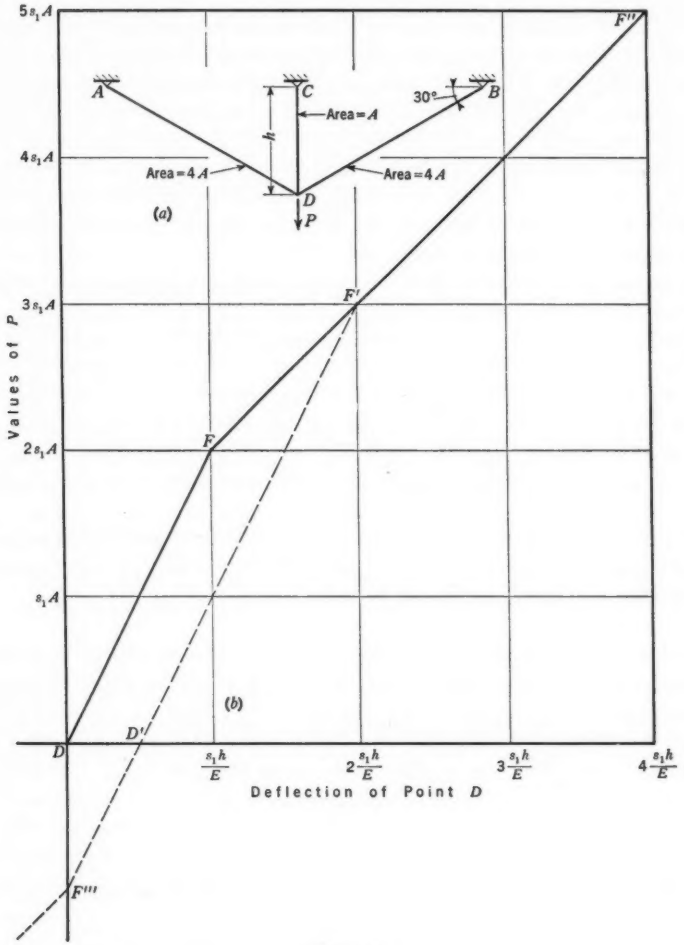

FIG. 18

of ultimate strength to working stress. The idea of stress is intimately connected with this definition. A "factor of safety" based on the ratio of yield stress to working stress is connected with the definition of yield stress which is itself defined in terms of deformation.

The author is to be commended for bringing this subject to the attention of the profession and for his success in clarifying many of the salient points.

EDWARD GODFREY,[23] M. AM. SOC. C. E. (by letter).[23a]—The subject of this excellent paper is one that has been neglected or ignored to a large degree by writers on the theory of elastic members. Much has been written on the behavior of materials that are perfectly elastic under the action of stresses, but as there are scarcely any materials that approach this quality there is a hiatus between the theory and the actual fact as represented by the materials under stress. A perfectly elastic material, of which glass is the best example, would be exceedingly dangerous, in many situations, as a structural material.

Professor Van den Broek has cited some cases where the hiatus referred to is most conspicuous. Riveted joints, for example, furnish an example in which the assumption of perfect elasticity would develop, theoretically, a case of certain failure; but failures in riveted joints, where proper rules of design are followed, are totally unknown, and the rules of design virtually ignore perfect elasticity. It is only because structural materials are ductile as well as elastic that they adjust themselves to conditions, and riveted joints designed accordingly, and according to good rules, are safe.

The paper also refers to bends in parts that are under high stress, bends that are made on steel when it is cold and that stress the steel beyond the elastic limit. When the bend is not reversed, the steel is safe for unit stresses that would be considered proper in steel that is absolutely straight.

The entire fabric of the theory of secondary stresses, the "Theory of Limit" or, more properly, the "Limit of Theory," falls and fails when it is applied to such cases as stresses due to deflection of rigid structures, stresses due to rotation of members on pins, caused by deflection of a truss, and some other cases.

One of the safest structures in existence is the Firth of Forth tubular bridge, in Scotland, although computed secondary stresses, based on the elastic theory, would show disastrously high unit stresses.

BASIL SOUROCHNIKOFF,[24] ESQ. (by letter).[24a]—In the theory of limit design Professor Van den Broek assumes that immediately after a material has reached the elastic yield point, a perfect ductile flow occurs; that is, the material suffers considerable additional deformation, the stresses remaining constant. The stress-strain diagram is composed of two straight lines: Line $A\ B$ corresponds to the elastic phase; and, Line $B\ C$, parallel to the strain, corresponds to the ductile phase (see Fig. 2). This hypothesis is very interesting. The actual material will probably follow a curve between Line $B\ C$ and the extension of Line $A\ B$ so that the designer can obtain a quite clear insight into the behavior of the structure. Line $B\ C$ is probably limited by some Point C where actual rupture occurs after the material has undergone a certain ductile deformation.

The following analysis of a built-in beam, similar to the one analyzed in Case 1 by the writer, will serve to demonstrate the limitations of application of the theory. Suppose a straight beam $A\ B$ of Span L, connected at Ends A

[23] Structural Engr., Pittsburgh, Pa.
[23a] Received by the Secretary March 14, 1939.
[24] Structural Engr., St. Paul, Minn.
[24a] Received by the Secretary March 16, 1939.

and B into rigid supports, and carrying two symmetrical concentrated loads P at Points C and D at distances a from the ends A and B respectively. In this type of loading the moment curve is composed of straight lines, and therefore the algebraic calculations are quite simple.

Increase the loads P until the elastic yield point is reached at the supports. The bending moments at the supports (M_0) is then expressed by Equation (1) if s = the yield stress, I = moment of inertia, and c = the distance of the extreme fiber from the neutral axis. The moment at the center is less than M_0. If the loads P are further increased, the moment at the center increases. One must assume that the moments at supports also increase somewhat, and that the ductile phase is extended over lengths $A\,E$ and $B\,F = l$ at each end of the beam. Otherwise, a finite angular deformation would have to occur in an infinitely small length of the beam, which is improbable (see text preceding Equation (2)).

The loads P are now increased until the moment in the central part almost reaches the value M_0 as defined herein. The moments at the supports are then $\kappa\,M_0$, κ being a coefficient greater than unity. The lengths at each end of the beam where ductile flow occurs, can be calculated easily by noting that the moment diagram is a straight line, and that at Ends E and F of the lengths l the moment is M_0; thus:

$$\overline{A\,E} = \overline{B\,F} = l = a\,\frac{\kappa - 1}{\kappa + 1} \qquad (6)$$

In the region $E\,F$ the material is completely in an elastic state and it is easy to find the slope α of the deformed axis of the beam with the horizontal at Points E and F; thus:

$$\alpha = \frac{M_0}{E\,I}\left(\frac{L}{2} - a\right) = \frac{s}{E}\,\frac{b}{c} \qquad (7)$$

in which $b = \frac{L}{2} - a$. The change of slope in each of the lengths l must be arithmetically equal to α as the beam is built into rigid supports. The average change of slope, per unit length, in the region where ductile flow occurs is, therefore,

$$\frac{\alpha}{l} = \frac{s}{E}\,\frac{b}{c\,a}\,\frac{\kappa + 1}{\kappa - 1} \qquad (8)$$

At the points when the unit change of slope is the greatest its value α_M is obviously greater than the average, so that $\alpha_M > \frac{s}{E}\,\frac{b}{c\,a}\,\frac{\kappa + 1}{\kappa - 1}$. Assuming that κ is greater than one, the minimum value of α_M is obtained by selecting the greatest possible value of κ. For a rolled beam, 12 in. by 12 in. by 65 lb, the greatest possible value of κ is 1.0662 (see last line of Case 1) and therefore the smallest value of $\frac{\kappa + 1}{\kappa - 1}$ is about 30. Consequently, $\alpha_M > 30\,\frac{s}{E}\,\frac{b}{c\,a}$. Now, independently of the system of loading, the unit change of slope at any point cannot be greater than a certain value α_x, which is dependent on the properties of the material and of the cross-section. The length of an extreme compressed

fiber, originally equal to one, cannot be reduced to less than zero. The length of the extreme tension fiber, originally equal to one, cannot be stretched beyond a certain value u before a break occurs. Therefore, $\alpha_x < \frac{u}{2c}$. If, therefore, there is a ductile flow, but no actual break of fibers occurs in any place, one must have: $\alpha_M < \alpha_x$; therefore, $30 \frac{s}{E} \frac{b}{c\,a} < \frac{u}{2c}$; and, $\frac{u\,E}{s} > 60 \frac{b}{a}$.

Symbols u, E and s depend on the properties of the material only; b, a, c on the dimension of the beam and the positions of loads only. For a given material the value of $\frac{u\,E}{s}$ is given, and it seems, therefore, that (theoretically at least) one can determine a certain position of the loads for which, as the load is increased, a break at the supports will occur before the yield point is reached at the middle.

Reference to questions of this type was probably made when the author stated that he intentionally omitted studies of certain secondary considerations, thus diverting attention from the essential arguments (see text preceding Equation (2)).

Of course, if a few extreme fibers are broken at the supports, it does not mean that the entire beam will collapse; but permanent cracks will be formed and may be easily aggravated by frost, etc. The resistance to shear may also be impaired.

E. MIRABELLI,[25] M. AM. SOC. C. E. (by letter).[25a]—As applied to flexural members, the author's theory of limit design appears to provide an ample degree of safety. The theory is based on the conservative assumption that the yield point in bending is reached when the outer fibers begin to yield. Actually, beam deflection does not increase rapidly with a small increment of load until there is a considerable excess in the loading beyond that which causes yielding in the outer fibers. The load-deflection curve for a simple beam continues as a straight line (or very nearly a straight line) within the order of magnitude of elastic deflections during the period of time that the plastically stressed region in the beam extends from the outer fibers toward the neutral axis. Eventually, a loading is reached at which the beam deflection increases rapidly. The amount of this loading depends on the shape and proportions of the beam. These facts have been demonstrated mathematically elsewhere.[26] For these reasons the actual amount of the margin of safety exceeds the assumed amount and the theory of limit design seems justifiable when applied to beams.

However, the new theory does not seem to be so easily acceptable when applied to frames which involve direct stress. In connection with indeterminate trusses, the theory is that any over-stressed member of the frame will continue to carry its share of the load at a constant capacity, while the member is being distorted until the capacity load is fully applied to the frame. A

[25] Associate Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[25a] Received by the Secretary March 20, 1939.

[26] "Structural Application of Steel and Light-Weight Alloys: A Symposium," *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 1397–1403.

tension member may act in such a manner, but it is doubtful that a compression member is capable of carrying a constant amount of load after the yield point of the material is reached. An over-stressed compression member is likely to have a total resistance less than its capacity at the yield point (if it does not buckle and fail completely). This means that the other members of the frame must carry more than their design load. A reduction in the margin of safety is involved thereby.

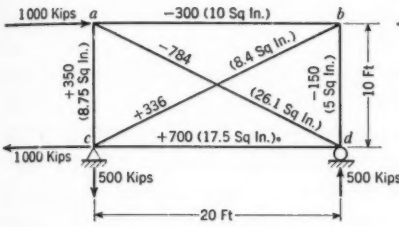

FIG. 19.—STRESSES BY LIMIT DESIGN THEORY OR BY ELASTIC THEORY

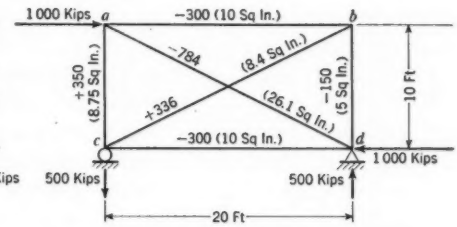

FIG. 20.—STRESSES BY LIMIT DESIGN

In the type of indeterminate truss shown in Fig. 19 any stress may be assigned to any member (such as − 300 kips to Member *a b*) and the stresses in the remaining members may be determined by application of the static conditions of equilibrium at successive joints. This is essentially the procedure of the theory of limit design. If, now, the required cross-section areas are determined from these stresses and the truss is analyzed by an elastic theory, it is found that the bar stresses are the same as those determined by application of the limit design theory. For such a case the two theories lead to the same design. The only difference is that the use of the new theory involves some saving in the time required for making the analysis.

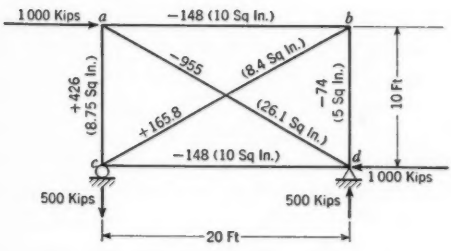

FIG. 21.—STRESSES BY ELASTIC THEORY

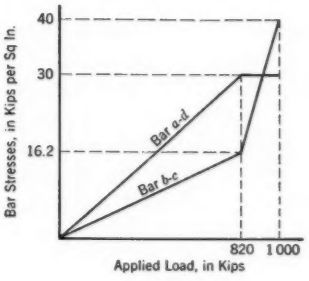

FIG. 22

Now, if the type of structure is changed by interchanging the hinged and roller supports, the elastic analysis and limit design analysis will not result in the same bar stresses. The stresses obtained by the limit-design method are shown in Fig. 20 and those found by the elastic method are shown in Fig. 21. The cross-section areas of members are assumed the same for the two cases. Diagonal *a d* will attain its capacity of − 784 kips when the applied load is

$1\,000 \times \frac{784}{955} = 820$ kips. Until this loading the stresses are within the elastic range throughout the frame, and at this loading the stress in Diagonal $b\,c$ is $165.8 \times \frac{820}{1\,000} = +\,136.0$ kips. According to the new theory, Member $a\,d$ will continue to offer its capacity resistance of $-\,784$ kips while the applied load is increased from 820 kips to 1 000 kips; and, the stress in Diagonal $b\,c$ is increased from $+\,136.0$ kips to $+\,336.0$ kips. This theoretical action is shown graphically in Fig. 22, in which stress intensities are used instead of total stresses.

The writer feels that there is no justification for assuming that the $a\,d$-line (Fig. 22) will continue horizontally at the level of 30 000 lb per sq in. Probably, there is more reason to believe that at the applied load of 820 kips the $a\,d$-line will reverse its direction and dip, or that Member $a\,d$ will be completely destroyed by buckling. As a consequence, the assumed margin of safety will be reduced, although it is true that in the majority of cases it is not likely that failure will occur at the working loads.

Possibly such objections could be overcome by use of a larger factor of safety for compression members than for tension members. However, if this were done, the economy in material effected by the application of the limit design method would be neutralized to some extent. The principal virtue of the new method, when applied to trusses, would then be in the simplification of stress analysis.

C. M. GOODRICH,[27] M. AM. SOC. C. E. (by letter).[27a]—Clearly and cogently, Professor Van den Broek has directed attention to the unfortunate change that the teachers, and the present generation in general, have made in Hooke's law, from *ut tensio sic vis* (as strain, so stress), to *ut vis sic tensio*. It is equally clear from his paper that no fault is found with any elastic theory in its proper field—namely, within elastic limits. He directs attention merely to the undoubted fact that there is another field, and one which may be of use to the engineer. In the later theories of combined stresses, strains are the foundation of the theories. In the recent analysis of welds, stress distribution has been considered to be represented by two rectangles, instead of by two triangles. Those who deal constantly with cases involving the same philosophy recognize the facts but usually do not discuss them. Holland, and some European cities, such as Hamburg and Budapest, give limit-design principles in their specifications, allowing elastic computation to those who prefer it.

The late F. C. McMath, M. Am. Soc. C. E., told the writer long ago that, in any structure, if any adequate path is provided to enable forces to reach the ground, any other path or paths taken will lessen work required of the structure as a whole. In one standard reference book[28] appears the statement, "These proximate solutions are offered as illustrative of simple approximate methods which may often be applied to very complicated problems of this class." In

[27] Chf. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[27a] Received by the Secretary March 2, 1939.

[28] "Materials of Construction," by J. B. Johnson, Fifth Edition, John Wiley & Sons, 1918, p. 36.

other words, this author recognizes the principles of what Professor Van den Broek has aptly called "limit design."

In a German work,[29] Föppl describes the treatment of structures over a street to protect traffic from falling objects, including this comment: "Accordingly, it is sufficient to equate the energy of the falling mass (after deduction of the impact-loss), with the form-changing work that must be expended in bending the beam severely, and it is sufficient to require that the impact loss does not exceed a reasonable fraction of the work expended." In addition, Föppl refers to A. Senft,[30] who apparently treats the subject more fully. He then cites the further case of bomb shelters, where also, clearly enough, limit-design principles were used. It is unfortunate that the logic of the case was not developed to show its many possibilities.

In 1910 designs for transmission towers were prepared by the writer's Company, using limit-design ideas freely. These designs, doubtless because of their unconventionality, were condemned by a succession of five different eminent authorities as inadequate to sustain the required load. Two sample structures, built to these designs and tested at the expense of the company, showed a load capacity about 50% in excess of that required as a minimum. Structures containing more than 7 000 tons, built under one contract to these and to similar designs and employing such methods as are advocated by Professor Van den Broek, are in service to-day. Even yet such steel is often sold under a guarantee that the structures will carry the required loads under test, usually plus some fixed percentage. This guarantee obviates a not improbable condemnation should the designs themselves be submitted for approval.

The logic of limit design is inescapable; its application is simple; and it promotes economy. In such a field as that of rivet groups under alternating stresses it exposes a danger not discussed in current textbooks, and not at all adequately explored in testing laboratories. These principles are under a kind of tabu, apparently, although they are familiar enough in some of their practical expressions. To determine their field of usefulness makes it necessary to open the windows of the mind, and to encourage a frank discussion of "whens" and "whys."

GEORGE WINTER,[31] ESQ. (by letter).[31a]—The writer has known the "theory of limit design" ever since Bleich[32] in 1932 developed a method similar to that proposed by the author. The fundamental principles underlying both of these methods are indeed extremely appealing to engineering common sense. Professor Van den Broek is to be commended for introducing this topic for discussion and thus opening new prospects for a more exact theory of design based on the criterion of permissible deformation.

However, in the author's treatment (as well as in Bleich's) it is precisely the question of deformation which, in the last analysis, remains unsolved.

[29] "Vorlesungen über die Mechanik," by August Föppl, Ninth Edition, Vol. III, 1921, p. 190.

[30] "Centralblatt der Bauverwaltung," by A. Senft, 1915, p. 233.

[31] Research Investigator, School of Civ. Eng., Cornell Univ., Ithaca, N. Y.

[31a] Received by the Secretary March 17, 1939.

[32] "Stahlhochbauten," by Hans Bleich, Berlin, 1932, Vol. 1, pp. 396–411.

Any design theory that does not provide the means of determining the deformations corresponding to any given load up to the ultimate one is incomplete and questionable. It is the indisputable strength of the theory of elasticity that it provides these means with a high degree of accuracy. Any design theory extending beyond the elastic range should attempt, at least, to satisfy this same condition, and even more so a theory which consciously assumes as its base the criterion of permissible deformation. This is not the case either in the author's theory or in Bleich's (the only one which formerly was known to the writer).

Realizing this deficiency the writer, in November, 1938, undertook to investigate the bending of steel beams beyond the elastic limit with the main purpose of its application to what the author, very exactly, calls "limit design." The writer intends to discuss herein only those aspects which bear a direct relation to the theory advanced for discussion by Professor Van den Broek. The most important question to be solved seems to be that of determining the deflection of beams, thus "over-stressed," in a manner similar to the determination of purely elastic deflections.

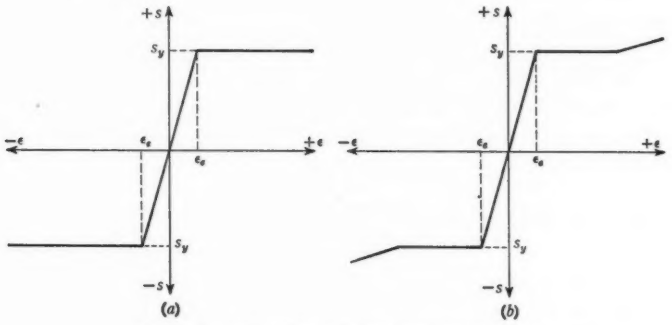

FIG. 23.—IDEALIZED STRESS-STRAIN CURVES

For his investigation the writer has used the same type of idealized stress-strain diagram Fig. 23(a) that underlies the author's and Bleich's theories as well as other treatments of plastic bending.[33] Fig. 24(b) represents the stress distribution over a cross-section after the elastic limit has been exceeded. At a distance $0.5\,q$ above and below the neutral axis the magnitude of the unit fiber strain is ϵ_e. Hence the differential equation of the bent axis of such a beam is

$$\frac{d^2y}{dx^2} = -\frac{2\,\epsilon_e}{q} \qquad (9)$$

in which $\frac{d^2y}{dx^2}$ is the curvature of the axis, ϵ_e the elastic limit strain (see Fig. 23(a)) and q the depth of the part of the cross-section within which the stresses are less than the yield point s_y—and, hence, where they obey the straight-line law (see Fig. 24(b)). As usual it is assumed that plane cross-sections remain

[33] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., 1931; and "Strength of Materials," by S. Timoshenko, Vol. 1, D. Van Nostrand, 1930.

plane if subject to this stress distribution, a fact which has been well confirmed experimentally in the plastic, as well as the elastic, range.[33] Since the moment of the external forces must be balanced by that produced by the stress dis-

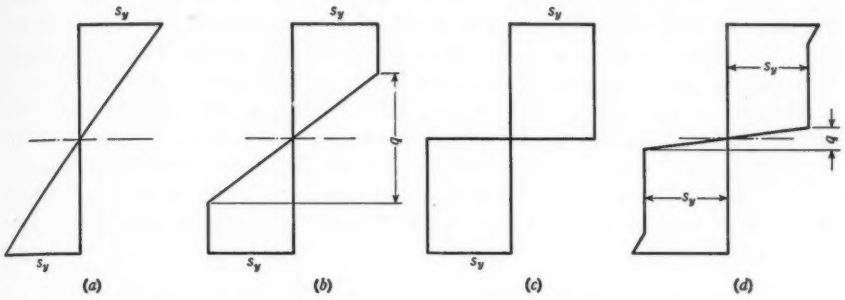

FIG. 24.—CONSECUTIVE STRESS DISTRIBUTIONS IN BENDING BEYOND THE YIELD POINT

tribution shown in Fig. 24(b), a simple equation of equilibrium determines q for any given moment. In this manner the writer has found for rectangular cross-sections

$$M = \frac{s_y}{4} b \left(h^2 - \frac{q^2}{3} \right) \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and for I-beams (see Fig. 25):

If $q \leqq h_1$,

$$M = s_y \left[b\, d_1 h_2 + \frac{d_2}{4} (h_1^2 - q^2) + \frac{d_2 q^2}{6} \right] \ldots\ldots\ldots\ldots (11a)$$

and, if $q \geqq h_1$,

$$M = s_y \left[b \frac{h^2 - q^2}{4} + \frac{b\, q^3 - (b - d_2)\, h_1^3}{6\, q} \right] \ldots\ldots\ldots\ldots (11b)$$

Obviously Equations (10) and (11) hold as long as the stress distribution of Fig. 24(b) holds and hence can be evaluated as long as $q \neq 0$. On the basis of the stress-strain curve of Fig. 23(a) q becomes zero when the moment of the external forces becomes equal to the internal moment corresponding to the stress distribution of Fig. 24(c). It is obvious that in this case

$$M = s_y S_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which the section modulus S_p is twice the moment of the area of the upper or lower half of the cross-section about the neutral axis; S_p will be called the plastic section modulus as compared with the usual elastic section modulus S_e. The ratio $S_p : S_e$ depends largely upon the shape of the cross-section; it is 1.5 for rectangular beams, 1.136 to 1.195 for standard I-beams and 1.07 to 1.175 for wide flange sections. If the moment of the external forces at any section reaches the value given by Equation (12), this section has reached its ultimate bearing capacity. It is transformed into a "plastic hinge"; that is, the two parts of the beam at both sides of this cross-section can be rotated with respect to each other through a considerable angle without increasing the load, the internal moment remaining unchanged during this rotation.

It may be noted that, actually, the state of an ideal "plastic hinge" can be approached but can never be reached completely, the reason being that the curve in Fig. 23(a) is idealized and for more accuracy should be replaced by Fig. 23(b) which includes work hardening. Then, instead of the stress distribution in Fig. 24(c), that in Fig. 24(d) will occur, in which q can never be zero, although it becomes very small for ordinary ductile structural steel. However, it may be shown from Equations (9) to (11) (as well as from tests performed by the writer) that the difference between the idealized and the actual state is negligible. In fact the difference between the moment of Distribution 24(c) and the actual moment at which Distribution 24(b) changes to Distribution 24(d) does not exceed a fraction of 1% and, from this load on the section, may be regarded as a "plastic hinge" with about the same degree of accuracy.

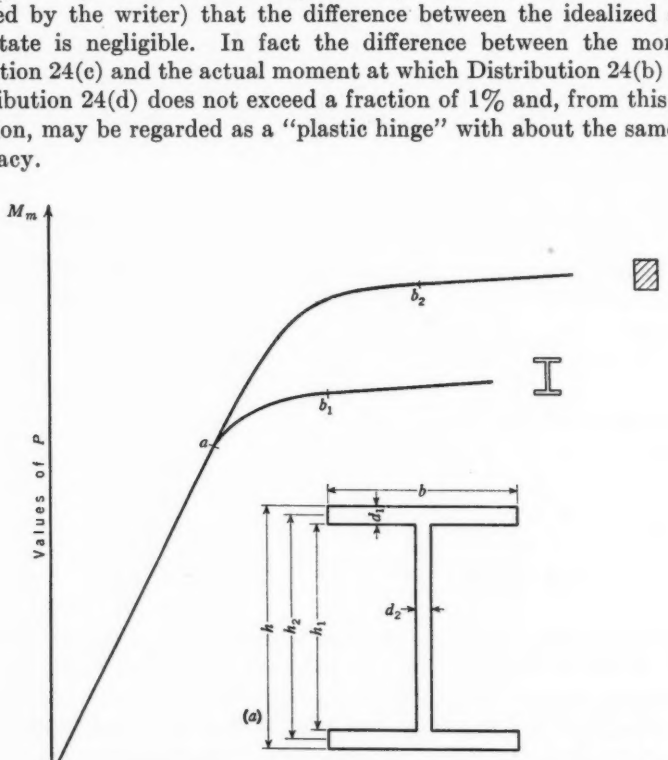

FIG. 25.—LOAD DEFLECTION CURVES OF A RECTANGULAR BEAM AND AN I-BEAM HAVING THE SAME SECTION MODULUS, S_e, AND THE SAME YIELD POINT, S_y

The writer has tested the foregoing ideas experimentally by means of bending tests performed on I-beams as well as rectangular beams. Although this investigation is not yet finished (1939), the results hitherto obtained fully confirm these principles. The general character of the load-deflection curves obtained is shown in Fig. 25. The curves represent the mid-span deflection of an I-beam and a rectangular beam both having the same section modulus S_e. Point a, Fig. 25, corresponds to the maximum moment $M_m = s_y S_e$—that is, to the ultimate elastic load, or moment, at which the transition from Stress Distribution 24(a) to Distribution 24(b) occurs; Points b correspond to

$M_m = s_y S_p$—that is, to the transition from Distribution 24(b) to Distribution 24(d), or to the moment at which a "plastic hinge" is formed. The experimental curves between Points a and b show good coincidence with the ones computed from Equations (9) to (11). The parts of the curves above Point b, Fig. 25, show that the idea of a plastic hinge is justified because from that point a very small load increment corresponds to a very large deflection increment. The importance of S_p is obvious from the curves.

The application of these principles to the "theory of limit design" reveals the following facts: In the author's Case 1 (on which the other cases are based so far as bending is concerned), the moment distribution of Fig. 26(a) is the ultimate one which is governed by the usual principles of the theory of elasticity and ordinarily is regarded as the limit so far as bearing capacity is concerned. Professor Van den Broek proposes to take the moment distribution of Fig. 26(b) as the ultimate, and to base the safety factor on this distribution. However, Distribution 26(a) is defined exactly; it actually occurs whereas it is easy to show that Distribution 26(b) never can occur. If, after having reached Distribution 26(a), the load increases further, the end moments increase as well as the mid-span moments. Whereas the latter still remain in the elastic range, however, the end moments exceed $M = s_y S_e = 30\,000 \times \frac{12}{12} = 30\,000$ lb-ft—in other words, Distribution 24(b) takes place. Regarding Fig. 25 as the curve of relative rotation of two consecutive cross-sections as plotted against M, the parts of the beam near the ends are in the section a-b of the curve, whereas the mid-span part is still in section o-a. Hence the deformations of the former are governed by Equation (9), and, those of the latter by the usual relation

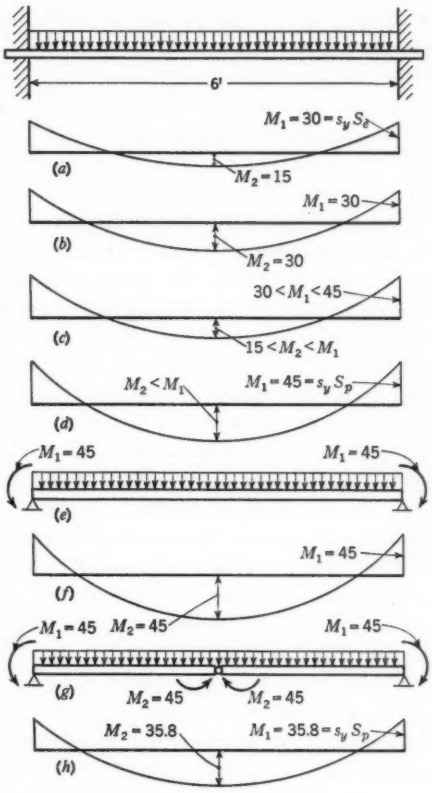

FIG. 26.—DISCUSSION OF CASE 1; MOMENTS ARE IN FOOT-KIPS

$$\frac{d^2y}{dx^2} = -\frac{M}{E\,I} \qquad (13)$$

The essential point is that by means of these two relations the entire state of deformation as well as of moment distribution is defined exactly in the same manner as it is defined in the elastic range by Equation (13) alone. On further

increasing the load, Moment M_1 finally reaches its ultimate value, $M = s_y S_p = 30\,000 \times \frac{18}{12} = 45\,000$ lb-ft (corresponding to Fig. 24(c) and Equation (12) and the end sections become transformed into "plastic hinges" (see Fig. 26(d)). The beam from now on acts as if it were freely hinged, with constant restraining moments at the supports (see Fig. 26(e)). In this state the moment distribution is determined by simple equilibrium relations and the deflections are still determinable by Equations (9) and (13). After a further increase of the load the mid-span section finally reaches its ultimate moment also, another plastic hinge forms at this point (see Figs. 26(f) and 26(g)) and, any slight further increase of the load results in excessive and uncontrollable deformation. In this entire process the deformations remain in the order of elastic deformations until shortly before the load corresponding to Fig. 26(f) is reached. (The foregoing idea is the one underlying Bleich's theory. The difference between his treatment and the one presented herein is that Bleich did not take into consideration the foregoing developed laws governing the deformations and moment distributions during the entire process. The method of integrating simultaneously the differential Equations (9) and (13) over the entire span is essentially the same as that used in the ordinary theory of elasticity for the integration of Equation (13) only. Moreover, the equations developed herein are still, fundamentally, elastic equations since the deformation is governed by the central part of the cross-section within q [see Fig. 24(b)] which remains elastic until the ultimate load is reached. However, the actual computation is somewhat cumbersome and exceeds the scope of this paper.)

As may now be seen, the moment distribution of Fig. 26(b), which underlies the author's concepts, actually cannot occur under any conditions. The author himself is well aware of this fact as may be concluded from his remarks at the end of the discussion of Case 1. However, the writer believes that, contrary to the author's opinion, these disagreements are far from being "secondary considerations." In principle it is scarcely possible to justify basing the safety factor on an entirely fictitious state such as Fig. 26(b), instead of on a clearly defined situation such as Fig. 26(a). The writer agrees fully with the author in that Fig. 26(a) by no means represents an ultimate load; but, if the safety factor or the working load is to be derived from the criterion of permissible deformation (as it should), the real and well-defined situation shown in Fig. 26(f) is by far preferable to the fictitious one in Fig. 26(b). In addition to these fundamental considerations important practical discrepancies result from the author's proposition, as may be shown by the following numerical example: Figs. 26(a), 26(b) and 26(f), respectively, correspond to loads of 10 000, 13 333 and 20 000 lb per ft. A safety factor of, say, 2 would bring the author's working load to 6 667, the writer's value to 10 000 lb per ft. (It should be emphasized that this value of the safety factor, the same as used by the author, is chosen purely for illustration and not as a practical design proposition, the question of finally deciding the actual value of the safety factor being rather complex. Offhand, a factor of 2 seems to be too low if applied to the ultimate load as defined in this paper. In the numerical example the factor

of 2, however, had to be maintained in order to obtain results consistent with the author's numerical example.)

Now, suppose that this same beam, instead of consisting of the rectangular bar 6 in. by 2 in., as in the author's example, is made of a standard I-beam 7 in. by $3\frac{3}{4}$ in. by 20 lb, which has the same section modulus $S_e = 12$. According to the author's proposition this I-beam would be attributed the same working load as the rectangular beam, namely 6 667 lb per ft. However, the actual moment transforming the M_m-sections into "plastic hinges" is $M_m = s_y S_p = 30\,000 \times \frac{14.31}{12} = 35\,800$ lb-ft. (For this I-beam, $S_p = 1.193\, S_e = 14.31$.) Hence, the moment distribution at which excessive deformation actually occurs is that of Fig. 26(h). The corresponding uniform load amounts to 15 920 lb per ft and, applying again the same safety factor, the working load would be 7 960 lb per ft. Hence, by the author's method, an actual safety factor of $\frac{20\,000}{6\,667} = 3$ would result for the rectangular beam as compared with $\frac{15\,920}{6\,667} = 2.39$ for the I-beam instead of the nominal safety factor of 2; whereas by the method proposed in this paper the actual safety factor in both cases would be the same, namely the factor selected. It should be emphasized that this definition of the safety factor or working load as related to the load which actually results in excessive deformation has a clearly defined meaning which makes it subject to exact mathematical analysis. The difference in both these concepts follows clearly from Fig. 25(a) which is obtained from actual tests and is in agreement with the theoretical considerations outlined. The author proposes to let both beams, the deflection curves of which are given in Fig. 25(a), carry the same allowable load, disregarding their difference in bearing capacity which is rather obvious from the curves. The writer proposes to determine the working load as depending upon the maximum moment M_m at which excessive deformation begins—that is, essentially, the moments corresponding to Points *b*. The computing work is exactly the same in both cases the only difference being that, for the writer's method, S_p must be known. For standard beams this value, as well as the usual S_e, could be tabulated.

The foregoing discrepancy results from the fundamental fact that once the elastic range of loading is exceeded, the usual section modulus S_e, as based on Fig. 24(a) loses its significance entirely and the only expression that characterizes the further behavior of the beam is S_p, corresponding to Fig. 24(c). The author's method is based entirely on S_e despite the fact that he deals with loads beyond the elastic range; and hence the discrepancies follow logically. The difference between the author's nominal and the actual safety factor would have been even greater if, instead of the I-beam—$S_p : S_e = 19.3$—a beam with, say, $S_p : S_e = 1.10$, had been analyzed.

The writer does not recommend applying the proposed method prematurely to actual design. Many questions must be clarified before a solid basis for a new design theory can be advanced. A number of these problems have presented themselves in the writer's present experimental investigation. For example, the relation between the yield points in bending and in tension, and the influence of the load or strain speed on the yield point seem rather problem-

atical. The questions of residual stresses in rolled sections and of dynamic loading are also so essential, with respect to "limit design," that they should be studied before a new design method is proposed for adoption. Wide possibilities for research are open in this field. It may be noted that similar tendencies toward limit design have gained ground in the field of reinforced concrete beginning from the time in 1931 when Emperger raised the question as to the actual meaning of the ratio n. Extensive experimental investigations have been conducted ever since that time and they have not begun to crystallize into a new design theory until now.

The author's important contribution is that he has clearly stated the problem of limit design, thus opening a discussion which, it is to be hoped, will result in useful new ideas in design. Decidedly, the writer shares the author's belief that, after sufficient additional knowledge has been obtained as to the fundamentals, the old design methods, considering the elastic range only, will indeed give way to a broader theory. Nevertheless, they will retain the methods of the theory of elasticity, but will be broadened to include the exact analysis of the actual ultimate strength as defined by permissible deformation.

FRANCIS E. SIMPSON,[34] M. AM. SOC. C. E. (by letter).[34a]—For its pioneering properties, this paper is a valuable contribution to the profession. It should stir the practicing engineer into a realization that perhaps he has become a slave to thinking mechanically in terms of working stresses without giving much heed to either elastic or ductile performance for either structures or parts of structures.

One cannot read this paper critically without becoming fully alive to the fact that, with shifting points of contraflexure and with fixity at the supports being altered in the process of ductile loadings, a beam loaded beyond its elastic limit becomes a most interesting subject of study with all kinds of possibilities in its elastic behavior.

The treatment of frames and of riveted joints brings out many points that are of prime importance, although some of these points are not given the weight they should be given by the majority of practicing engineers. The author states that the practicing engineer seems to have a dread of using stresses approaching the elastic limit, and possibly this is largely true. However, there are instances in which, during construction, parts of structures might necessarily be subjected to rather extreme conditions of loading, and in which designing on the basis of accepted working stresses would result in an uneconomical structure. It is fairly common in such instances for practicing engineers to use working stresses somewhat higher than those ordinarily prescribed. However, even if construction conditions would warrant the use of higher working stresses than is usual, the structure would be designed to carry its design loading with accepted working stresses.

In conclusion, it is appropriate to say that the author should be complimented not only for his splendid paper but for his daring in the suggestion of, what might be termed by some, a slight schism in the realm of thought concerning the theory of elasticity.

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[34a] Received by the Secretary April 10, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PRACTICAL METHODS OF RE-ZONING URBAN AREAS

Discussion

BY HUGH E. YOUNG

HUGH E. YOUNG,[11] M. AM. SOC. C. E. (by letter).[11a]—Data from seventy-five American cities with populations of 50 000 and more were contained in the paper. This material indicates that many people interested in the municipal problems of these cities are well aware of the need for re-zoning most of the communities. Several basic reasons were given why such a need exists. One was the failure of those who prepared the zoning ordinances to realize that zoning is fundamentally an element of city planning because it is the process which co-ordinates the social, economic, and physical structure of urban areas.

As Mr. Downs states, many zoning plans are merely maps of existing uses, and show no evidence of any attempt to co-ordinate zoning with city planning, or even to proportion the urban area between the various uses in an orderly or scientific manner. Failure to plan properly first, or to co-ordinate zoning with existing city plans, is perhaps one of the basic reasons why re-zoning is now necessary in so many large American cities, although it is not so vital in smaller communities.

Another reason was the illogical and inequitable allocation of land as between various uses, combined with improper distribution of the areas devoted to specific uses. As stated by Mr. Bartholomew, improper proportioning of land among the various uses is indicated by the fact that many cities have zoned an extensive area of land for one use or another that is unjustifiable in the light of well-known factors as to the area that is actually required in terms of acres per one hundred persons, and as to the annual rate of land absorption. Improper distribution (Mr. Bartholomew further states) is shown by the failure of many zoning ordinances to provide residential areas in appropriate relationship to places of work and business.

Mr. Kohn emphasizes a further reason outlined in the paper; namely, that

NOTE.—The paper by Hugh E. Young, M. Am. Soc. C. E., was presented at the meeting of the City Planning Division, New York, N. Y., January 21, 1937, and published in March, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1938, by Messrs. Robert D. Kohn, Myron D. Downs, Harland Bartholomew, and Benjamin Saltzman; and October, 1938, by G. H. Herrold, M. Am. Soc. C. E.

[11] Chf. Engr., Chicago Plan Comm., Chicago, Ill.

[11a] Received by the Secretary April 10, 1939.

much zoning is based on an incorrect assumption as to what constitutes the highest and best use of a given area, which, in turn, results in unduly high land values, or price as Mr. Kohn calls it. He points out that many zoning ordinances provide for a use in a given area that is speculative rather than factual, and for a density of use so intense as to make the assumed value of the land, if devoted to that use, far greater than its real value, if devoted to its genuine best use. Mr. Kohn believes that not only zoning, but city planning and housing progress as well, are largely dependent upon making the public and the real estate owner realize the difference between price based upon unrealistic zoning and real value based upon appropriate land use.

The paper advocated a real property inventory and land-use survey as the initial step toward re-zoning. The purpose of the survey is to show existing patterns of land use, areas of blight, population changes and other factors which together make re-zoning necessary. Mr. Bartholomew stresses the point that cities will be re-zoned to best advantage only when the re-zoning plan is based upon an accurate knowledge of the total land in urban use for various purposes and upon the annual rate of land absorption. He states that well-defined laws of land use, when thoroughly understood, will make possible far more scientific zoning practices than those of the past, and illustrates his point by giving the percentages of land used for various purposes (see Table 6): (1) As the average of sixteen self-contained cities; (2) in the St. Louis Metropolitan District; and (3) in the City of St. Louis alone. Table 6 indicates the importance of including all the area in urban land-use surveys, regardless of municipal boundaries, because the percentages for the City of St. Louis alone show certain marked variations from the normal averages.

A considerable part of the paper was devoted to the administration of a zoning ordinance and means for maintaining the integrity of the ordinance, listing some of the bad effects of allowing improper variations therefrom. The writer showed that no zoning ordinance will be effective unless its administration is in strong hands, free from political domination; and suggested that the essential need was for a strong public body, such as a plan commission, to lend support to the Zoning Board of Appeals. Furthermore, he showed that a certificate of occupancy is most important for the proper functioning of the zoning ordinance, but that such certificate should not be issued by the same department that is charged with its enforcement.

Mr. Herrold asserts that zoning is a part of city planning and can best be interpreted and administered by planning-minded people. He adds that in St. Paul the Zoning Board created by ordinance is composed of the five members of the Zoning Committee of the City Planning Board of St. Paul.

Mr. Saltzman states that re-zoning, alone, is not sufficient to correct all the negative factors that contribute to the ineffectiveness of a zoning ordinance. He states further that, even with a good re-zoning plan, the ordinance must be administered honestly and capably if it is to be successful. Certainly, honest and capable administration of the ordinance is the keystone of the structure of zoning; without it any zoning or re-zoning activity is an idle gesture.

Mr. Saltzman states that the burden of administration and enforcement of the zoning ordinance will be lightened by the requirement of a certificate of

occupancy for every new or altered building. This is because the issuance of this certificate is an assurance that the building complies with all the laws applicable thereto, including the zoning ordinance, and occupancy contrary to the terms of the certificate is in violation of the law. Certificates are a public record, so that illegal occupancies may be noted and corrected. Mr. Saltzman, however, is firmly of the opinion that the department charged with the enforcement of the ordinance should issue the certificate, because any other method would result in a shifting of responsibility, and bickering between the agency enforcing the law and the one issuing the certificate.

Mr. Downs emphasizes likewise the need of a strong administrative branch of government if good zoning is to succeed. Mr. Bartholomew advocates an even more advanced step than the writer's organization of a strong citizen's committee in order to preserve the integrity of a re-zoning ordinance. He suggests the establishment of permanent agencies in the form of neighborhood unit organizations in the interest of slum clearance, blighted district rehabilitation, and the protection of ideal residential areas. His point is that uncontrolled speculation in urban real estate has destroyed the stability and desirability of home ownership; hence it is essential to control real estate in a manner consistent with normal land use. Mr. Bartholomew, therefore, favors having the plan commission determine the broad principles of land policy, and having the citizen's committee as recommended; but in addition he recommends the establishment of well-organized neighborhood units to assist in adopting, enforcing, and maintaining appropriate zoning policies and a sound land program.

One defect in present zoning practice to which the writer called attention was the lack of adequate "buffer" areas between various uses. He stated that zoning practice is still in a crude stage with respect to the "tapering off" process—that is, the transition from a higher to a lower use, especially by means of park strips between residential sections and those used for other purposes.

Mr. Herrold agrees that suitable buffering must be considered, and that there must be a natural stepping up of uses from the less restricted districts to those which are more highly restricted. He gives it as his opinion that the use of parkways for buffering industrial districts is probably one of the best methods.

The paper demonstrated the detrimental effect of zoning variations and the problem of non-conforming use. Mr. Saltzman states that the full benefits of zoning or re-zoning cannot be secured unless the concomitant problem of non-conforming use is solved. He calls attention to the fact that any re-zoning will result in there being, in most districts, many buildings whose use does not conform to the regulations for the new district. Although obsolescence and natural decay will force the eventual demolition of some of these structures, this process will be hampered, the effectiveness of the ordinance will be impaired and the new restrictions will be valueless if the authorities allow indiscriminate variances that permit of the modernization or enlargement of such structures.

It is naturally gratifying to find so complete an agreement on the part of all those who participated in the discussion with respect to the principles set forth in the paper. One point made, however, was not discussed by any one.

It is a matter of such basic importance that it should, once again, be called to the attention of the reader. That is the need, when re-zoning, to provide for reciprocal protection for all the various use districts.

Every one is aware of the need for protecting residential areas against the incursion of commerce and industry, but it is equally important, for example, to protect manufacturing areas against incompatible commercial uses, and particularly against the restrictions which often result from the presence of near-by residential use—restrictions which work a hardship upon manufacturing operations of the so-called "nuisance" type but which are, nevertheless, an important element in the city's business and financial life.

Much of the paper was devoted to the subject of blighted areas and the possibility of their redemption. Re-zoning was suggested as the first step, apportioning city areas for various uses in accordance with definite factors of land-use absorption as developed by a land-use survey, because improper zoning is a blight in itself in that it specifies a use and volume that can never be attained.

The second suggested step was the preparation of an official plan by an official planning body, and the establishment of a strong citizen's group to advise the plan commission with respect to zoning, and to throw the weight of all local civic organizations back of the re-zoning plan.

The writer took the position that both the paternalistic (public) and the non-paternalistic (private) methods of providing adequate housing facilities were necessary elements in the rehabilitation of blighted districts. The paternalistic method is that of providing low-cost housing for the lowest income groups by governmental agencies and governmental financing. The non-paternalistic method, enabling groups of property owners to redeem a neighborhood as a private undertaking, would seem to have taken hold on the public mind in recent months. State legislation is pending in Illinois, and has been suggested elsewhere throughout the nation, looking toward the establishment of "Public Service Building Corporations" with specified powers of land condemnation under the right of eminent domain. This latter feature is designed to overcome what has been heretofore the greatest obstacle to the rehabilitation of blighted areas as a private enterprise.

The public service building corporation plan, originated in Chicago as a means of interesting private capital in slum clearance housing projects, is now (1939) being considered in New York City. The Merchants Association of that city has also suggested that additional legislation be enacted which will permit the formation of local district improvement corporations. In this plan the equity would be represented principally by shares of stock given by the corporation to present owners and mortgagees in exchange for their claims on the property and its present and potential earning power.

For both the public service building corporation and the local district improvement corporation, powers of eminent domain would be granted upon proof that they were necessary to bring minority groups or their property within the scope of an approved large-scale rehabilitation plan.